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## The factoring of large integers by the novel Castell-Fact-Algorithm, 12th part.

Following the introduction of the novel Tietken-Castell-Prime-Algorithm in order to definitely and correctly identifying and generating prime numbers of unlimited size in an indirect way.


#### Abstract

Our essays 1 to 11 describe the applicable Castell-Fact-Algorithm, which factorizes large integers, was ignored and rejected by economy and politics. Innovations concerning data protection and security seem not to be in great demand by neither the NSA, nor by Facebook \& Co. In the following essay part 12, we introduce the Tietken-Castell-Prime-Algorithm, which is able to indirectly a) produce prime numbers b) identify prime numbers, c) and which is suitable for the factoring of large numbers (composed of prime numbers) after creating a comprehensive registry of numbers.


## Objective

Following question would not appear, if for the factoring of large integers, large integers were given, which reliably emerged from the multiplication of two prime numbers.
It is debatable, whether the given "large number" is a product of two prime factors or a prime number itself (which can not be factored). Maybe it is even only product which ends with the digits $1,3,7$ or 9 . Therefore, a reliable method for clarification is needed in order to determine what exactly the given large number represents.
Taking a look into the many existing, non-reliable and ambiguously functioning methods of solving the issue, one would realize that a new approach without uncertain multiple trials and error rate (Fermat, Miller-Rabin, Chinese remainder theorem, Mersenne prime, etc.), as well as without Probability Theory, approximation, repeats, etc., is needed in order to obtain more than just a "most likely prime".
Even RSA (Ron Rivest, Shamir, Adleman) only utilizes "random numbers", which do not generate exact prime numbers but only "probable prime numbers".
The idea of the novel "Tietken-Castell-prime-Algorithm" saves the user the aforementioned uncertainties and ambiguities.
It delivers 100 \% prime numbers, which are definite and in all magnitudes.
It furthermore avoids the disadvantage, that big anterograde "skips" cause smaller prime numbers which are in between these skips to be overseen or omitted, which is the inconvenience of the RSA-multiplication of two identical prime numbers.
Shortly, it creates a dearrangement or disarray in the assumption of which other prime numbers do in fact exist.
If there would be total clearness and transparency, the RSA would not have multiply "probable prime numbers" with themselves for reasons of certainty.

## Personal preliminary remark concerning the "Tiekten-Castell-Prime-Algorithm"

The hereby presented novel algorithm is not only able to identify prime numbers in an indirect fashion, but also to generate them in that same fashion.
The solution suggested by the aforementioned novel procedure is simple and self-evident.
To generate a prime number, large enough to fill a dozen of folders, may be an impressive display of the quantitative potentials of current computers, but has nothing to do with the ever-growing qualitative human thinking (comparison to Gimps and the 400 years old Mersenne prime, etc.), apart from initial thoughts of an algorithm for the relatively small prime numbers in the beginning.
It is nothing but a currently still useless show with skyrocketing electricity bills and only a minimum of human in-house effort.

## Verbal explanation of the "Tiekten-Castell-Prime-Algorithm"

The fundamental consideration of the "Tiekten-Castell-Prime-Algorithm" is the currently still established impossibility to rapidly detect and generate large prime numbers, through an indirect manner.
a)

If prime numbers are not simply and correctly producible, one can use so called Pendants, which emerge from the differential method and are ideally (as shown here) exactly calculable.
These are the well-ordered, constant-remaining counting factors on one side and the two-step cardinally-growing counted factors of the also well-ordered and comprehensively growing nonprime numbers (or products in the close vicinity of the wanted prime numbers) on the other side. The prime numbers in this case are the ,remainders". They are numbers without factors!
b)

If prime numbers can be generated in this indirect fashion, then in a way that these prime numbers are identifiable accroding to their position.
Due to simplicity and the fact that all steps and procedures stay constant, there is an aligned order of prime numbers built up by their respective values, which exhibits no possible gaps or mistakes; meaning every prime number is captured and made visible.
Once put into motion, the Tietken-Castell-Prime-Algorithm is able to build up a constantly growing registry without any following input, which is ordered according to the scores of numbers and can be accessed at any time.
Thereby, no human interaction or effort is needed - the procedure of operation is automatized.

## Numerical examples of the "Tiekten-Castell-Prime-Algorithm"

The algorithm builds itself up only with the final digits $1,3,(5), 7$ and 9 per decade (every 10th). Number " 5 " is put in brackets, because it acts as an exception.
It may (except as a single digit) never become a prime number, even with added decimal digits, due to the fact that numbers with the final digit " -5 " are always divisible by 5.
However, the Tietken-Castell-System counts the " 5 " as well, in order to keep a consistent increment and to avoid the necessity of a double-step every time the final digit "-5" comes up, in which the " 5 " will be skipped.
The following graph shows how the Tietken-Castell-Registry grows in a constant-remaining fashion (score-wise). According to our premise of a constant-remaining decimal system, it could be continued indefinitely.
a)

In each of the indefinite amount of rows, only numbers with the final digits 1, 3, 5, 7 or 9 are being captured, because every prime number has to exhibit either 1, 3, 7 or 9 as a final digit (digit 5, as exception, is dropped).
However not every number with these final digits is a prime number.
So, in order to comprehensively capture all prime numbers without gaps, all numbers which exhibit one of the four prime-end-digits, have to be included into the registry as well. Among them also the prime numbers are to be found. These are identified by the fact that there are no existent factors for them.
b)

The rows within the Tiekten-Castell-Registry always look alike. Each row exhibits the final digits 1, 3,5, 7 and 9.
Every newly added row, receives an additional decade with 1, 3, (5), 7, 9 on the end.
13579
1113151719
2123252729
3133353739
4143454749
5153555759
6163656769
7173757779
etc.
or:
10011003100510071009
10111013101510171019
10211023102510271029
etc.
or:
23812383238523872389
23912393239523972399
24012403240524072409
24112413241524172419
etc.

## c)

The claim here is that there is no limitation of numbers towards the top!
However, not an approximately 2-million-digits prime number is looked for, but preliminary "only" "large numbers" of 1000- to 2000 digits, which emerged through multiplication of two prime numbers.
According to our estimation, this task (even with the consideration that prime numbers are getting rarer and rarer within the increasing numbers) could be executed easier and faster than going just by the vague procedure of finding prime numbers by chance, which are not even reliably identified or confirmed.

In order to do the task correctly the "Tiekten-Castell-Prime-Algorithm" has to count up without gaps, constantly add, as well as keep the sustained connection to each previous factor.

Furthermore it has to establish multiplications between two prime numbers and save the results as „large numbers" to recall them later with the respectively given factors!
d)

In order to only keep numbers with the final digits $1,3,7$ or 9 in the registry, all numbers with the final digits $0,2,4,6$ or 8 have to be excluded from the registry.
Final digit " 5 " is counted along in the registry, but plays no further role.
e)

Each number of the register functions and provides itself as a constant factor for the own respective row of following numbers (in distances of two times the respective number).
Every time a prime number emerges due to the lack of potential factors, it is also multiplied by itself and starts a new unlimited row of numbers in the registry, from the point of the emerged product (a large number from two prime numbers).
Prime numbers are numbers which occur without factors within the Tietken-Castell-Registry.
However, if there is an involvement of factors (two or more) in the formation of the given number, it is impossible to be a prime number.
Because the respective factors are past on the "left side", the product is calculated there already.
If this in the past calculated number shows up then again, it will already be known as the product together with its respective factors.
This way, it can also be determined whether the given number is a "large number", meaning a product of two prime numbers.

## f)

Following, the mechanism of action of numbers within the Tietken-Castell-Algorithm is exemplified.
Inevitably the origin starts with small numbers. Though, the principle of approach and utilization continues, as before indefinitely towards the top, because the laws of the decimal system remain the same for small and large numbers. Also the constant approach of the
Tietken-Castell-Prime-Algorithm remains unchanged.
g)

## 1. Row, 1. Number: 3

3 is (as all numbers of the registry) multiplied, makes 9 and begins the first row of numbers from here, which traverses through all, constantly increasing numbers in the registry.
This number 3 remains as a counting constant and builds up new numbers; first with itself and then with the uneven numbers $5,7,9,11,13,(15), 17,19,21,23,(25), 27,2931,33,(35), 37$, 39 etc., which sometimes may appear as normal products, but in other cases also happen to be products of two prime numbers.
3 is a prime number, because it has no factors "from the left" in the registry.
If 3 is multiplied by e.g. 11 or 13 (for which the same rules apply), the respectively emerged product of 33 or 39 acts as a "large number" suitable for encrypting.
With the increasingly frequent rows of numbers, which run indefinitely through the entire register, there can be multiple assignments of numbers, in places where two rows of numbers overlap. This means that the respective number has multiple factors.
Though, most importantly for the Tietken-Castell-Prime-Algorithm in this context, is to determine whether the number even has factors or not.
If so, it can be elicited, whether these were prime-factors and if the product is suitable as a
"large number" for Cryptography.
If the number has no factors, it is (as previously mentioned) a prime number itself. In which case there can not be a multiple assignment.
h)

## 1. Row, 2. Number: 5

5 is a prime number, similar to number 3, because there are no possible factors that could lead to it as result.

Its row of numbers starts from $5 * 3=15$, but even with added decimal digits it will never be a prime number again, due the fact that it is always divisible by 5 .
i)

1. Row, 3. Number: 7

7 is as well a prime number, starting its unlimited march through the Tietken-Castell-Registry with 7 * 3 ( = 21) as a 7-row.

## j)

## 1. Row, 4. Number: 9

Number 9 acts as an interstage of the 3-row and therefore not a prime number.
However, together with decimal digits, it builds an important prime-end-digit and often results in prime numbers itself (e.g. 19, 29, 59, 79, 89, 109, 139, 149, 179, 199, 229 239, 269, 349, 359, 379, 389, up to infinity!)
The 9-row, which will traverse the registry starts with $9 * 3(=27)$.
Together with number 9 as a constant factor and the other uneven factors which stay constant in all rows (besides 3, the $5,7,9,11,13,15,17,19$ and so on, indefinitely), the first numbers of the 9 -row a built-up.
The fact that the single digit " 9 " is not a prime number, prevents it from potentially creating "large numbers" used for Cryptography, even though the counted second factors 11, 13, 17, 19, 23, 29 and 31 are prime numbers and would have been suitable for it.
This initial situation though, changes its context with decimal digits. Already from the aforementioned 19, 29, 59, 79, 89, 109 etc. on, the prime-end-digit 9 is back in the race as a part of prime numbers.

## k)

## Résumé of the 1. Row:

Already with these four mentioned examples only, it is visible that the factors (given they exist) follow a regular order.
On one side are numbers which always have the same final digits in the same order (1, 3, (5), 7, 9 ), while on the other side the same uneven numbers are constantly and cardinally counted-up in double steps.
I)

## 2. Row, 1. - 5. Number: 11-9

The second row follows the same fashion of calculation as the first one (same as all indefinite, following numbers).
(1)

11 is multiplied buy $3,5,7,9,11$, etc.
After multiplication by itself, $11 * 11=121$ ( a "large number"), it continues its row with $11 * 13$, $11 * 15,11 * 17,11 * 19,11 * 21$ etc., up to infinity.
For the purpose of encryption, it would not be necessary to multiply a prime number like 11 by itself, because the other prime numbers to form a "large number" with are known.

13 is multiplied by $3,5,7,9,11,13$ etc.
After multiplication by itself, $13 * 13=169$ ( a "large number"), it continues its row with 13 * 15, $13 * 17,13 * 19,13 * 21,13 * 23$ etc., up to infinity.

For the purpose of encryption, it would not be necessary to multiply a prime number like 13 by itself, because the other prime numbers to form a "large number" with are known.

15 is multiplied by $3,5,7,9,11,13,15,17,19$ etc., but could never create a "large number" for the RSA-Cryptography, due to the fact of only being a prime number as a single digit (5).
(4)

17 is multiplied by $3,5,7,9,11,13,15,17$ etc.
After multiplication by itself, $17 * 17=289$ ( a "large number"), it continues its row with $17 * 18$, 17 * 19, 17 * 21, 17 * 22, 17 * 23 etc., up to infinity.
For the purpose of encryption, it would not be necessary to multiply a prime number like 17 by itself, because the other prime numbers to form a "large number" with are known.
(5)

19 is multiplied by $3,5,7,9,11,13,15,17,19$ etc.
After multiplication by itself, 19 * 19 = 361 ( a "large number"), it continues its row with 19 * 21 , $19 * 23,19 * 25,19 * 27,19 * 29$ etc., up to infinity.

For the purpose of encryption, it would not be necessary to multiply a prime number like 19 by itself, because the other prime numbers to form a "large number" with are known.
(6)

21 is multiplied by $3,5,7,9,11,13,15,17,19,21$ etc.
After multiplication by itself, $21 * 21=441$ ( a "large number"), it continues its row with 21 * 23, $21 * 25,21 * 27,21 * 29,21 * 31$ etc., up to infinity.
m)

## Résumé of the first to the indefinite "last" Row:

The above shown factors prove the order, predictability and correctness of this Tietken-Castell-Prime-Algorithm.

None of the previously mentioned unreliable and complicated methods were utilized in this algorithm for determenitation, whether a more or less randomly found- or unreliably calculated number is in fact a prime number.

If in one row, here for example the third row, where 21 and 27 can be seen, we know from the first row that 21 belongs to the 3 -Row (from $3 * 7$ or written in form of addition to $3+6+6+6$ ), as well as that 27 belongs to the 9 -Row (from $3 * 9$ or in form of addition $9+18$ ).
However, no factors lead to the numbers 23 and 29 of the same (here third) row.

## n)

## The Distances

It might be a relief for the algorithm, to only having the necessity to perform addition.
It is observed that the distances between all numbers in a row stay constant, due to the first multiplication by itself (meaning a multiplication by 2 ). It always builds exactly twice the first counting factor.
Insofar, the algorithm could (after the initial multiplication) build the following numbers in a row also via addition of the ever constant distances.
(1) 3 to 9 builds the distance 6 ; therefore the following numbers after this 9 are $15(9=6)$, 21 (=15+6), $27(21+6), 33,39,45,51,57,63,69,75$ and so on.
(2) 5 enlarges by 10 .-steps (from $2 * 5$ )
(3) The same principle accounts for number 7, where distances between the numbers in that row will be $2 * 7=14$ ( $49,63,77,91,105,119$, and so on).

## The beginning of the 3-Row:

In the following graph, the factors of all numbers are to be found on the right hand side, which are hinting towards the later product. The algorithm saves and stores these.

On the left hand side of those later products, their respective factors are noted once more in order to demonstrate the connection between them two.

As now known, prime numbers do not have such hints towards the factors on their left hand side.
Nevertheless, the absence of these left hand standing factors next to the numbers in this partial list (which only shows the progression of 3; so it is incomplete), does not give information about which kind of number we are looking at (prime number, "large number" or simple product).
Again, there are several multiple assignments of single numbers in the beginning of the registry, which means that rows are overlapping.
For example: crossing of the 3 -Row with $3 * 21$ and the 7 -Row with $7 * 9$ in the number " 63 ".
In the second case ( 7 * 9), the 63 would be suitable as a "large number".
(It has to be verified if such overlaps of multiple pairs of factors in one number would not increase the cryptographic certainty of a "large number", due to the fact that the factoring here is less obvious)

## A)

## The first 120 numbers, respecting only the 3-Row:

```
1 3(3*3=9) 5 (3*5=15) 7(3*7=21) (3*3=9)9(3*927)
11(3*11=33)13(3*13=39) (3*5=15)15(3*15=45) 17(3*17=51) 19(3*19=57)
(3*7=21)21(3*21=63) 23(3*23=63) 25(3*25=75)(3*9=27)27(3*27=81) 29(3*2=87)
31(3*31=93)(3*11=33)33(3*33=99)35(3*35=105)37(3*37=111)(3*13=39)39(3*117)
41(3*41=123) 43(3*43=129) (3*15=45)45(3*45=135) 47(3*47=141)49
(3*17=51)51(3*51=153) 53(3*53=159) 55(3*55=165) (3*19=57)57(3*57=171) 59(3*59=177)
61(3*61=183) (3*21=63)63(3*63=189) 65(3*65=195) 67(3*67=201) (3*23=69) 69(3*69=207)
71(3*71=213) 73 (3*73=219) (3*25=75)75(3*75=225) 77 (3*77=231) 79(3*79=237)
(3*27=81)81(3*81=243) 83(3*83=249) 85(3*85=255) (3*29=87)87(3*87=261) 89(3*89=267)
91(3*91=273) (3*31=93)93(3*93=279) 95(3*95=285) 97(3*97=291) (3*33=99)99(3*99=297)
101(3*101=303) 103(3*103=309) (3*35=105)105(3*105=315) 107(3*107=321)
109(3*109=327)
(3*37=111)111(3*111=333) 113(3*113=339) 115(3*115=345)
(3*117=351)117(3*117=351)119(3*119=357)
121(3*121=363)(3*41=123) 123(3*123=369) 125(3*125=375) 127(3*127=381)
(3*43=129)129(3*129=387)
131(3*131=393) 133(3*133=399) (3*45=135)135(3*135=405) 137(3*137=411)
139(3*139=417)
(3*47=141)141(3*141=423) 143(3*143=429)145(3*145=435) (3*49=147)147(3*147=441)
149(3*149=447)
```

B)

The first 120 numbers, respecting only the 5-Row:

## (No prime-end-digits, but uneven. Distances in 10.-steps)

```
1 3(5*3=15) 5(5*5=25) 7(5*7=35) 9(5*9=45)
11(5*11=55) 13(5*13=65) (5*3=15) 15(5*15=75) 17(5*17=85) 19(5*19=95)
21(5*21=105) 23(5*23=115) (5*5=25)25(5*25=125) 27(5*27=135)29(5*29=145)
```

```
31(5*31=155) 33(5*33=165) (5*7=35)35(5*35=175) 37(5*37=185) 39(5*39=195)
41(5*41=205)43(5*43=215) (5*9=45)45(5*45=225)47(5*47=235)49(5*49=245)
51(5*51=255) 53(5*53=265) (5*11=55)55(5*55=275) 57(5*57=285) 59(5*59=295)
61(5*61=305) (7*9=63)63(5*63=315) (5*13=65)65(5*65=325) 67(5*67=335) 69(5*69=345)
71(5*71=355) 73 (5*73=365) (5*15=75)75(5*75=375) (7*11=77)77(5*77=385)79(5*79=395)
81(5*81=405) 83(5*83=415) (5*17=85)85(5*85=425) (5*29=87)87(5*87=435) 89(5*89=445)
(7*13=91)91(5*91=455) 93(5*93=465) (5*19=95)95(5*95=475) 97(5*97=485) 99(5*99=495)
101(5*101=505) 103(5*103=515) (5*21=105)105(5*105=525) 107(5*107=535)
109(5*109=545)
111(5*111=555) 113(5*113=565) (5*23=115)115(5*115=575) 117(5*117=585)
119(5*119=595)
121(5*121=605) 123(5*123=615) (5*25=125)125(5*125=625) 127(5*127=635)
129(5*129=645)
131(5*131=655) 133(5*133=665) (5*27=135)135(5*135=675) 137(5*137=685)
139(5*139=695)
141(5*141=705) 143(5*143=715) (5*29=145)145(5*145=725) 147(5*147=735)
149(5*149=745)
```


## C)

The first 120 numbers, respecting only the 7-Row:

```
1 3(7*3=21) 5(7*5=35) 7(7*7=49) 9(7*9=63)
11(7*11=77) 13(7*13=91) 15(7*15=105) 17(7*17=119) 19(7*19=133)
(7*3=21)21(7*21=147) 23(7*23=161) 25(7*25=175) 27(7*27=189) 29(7*29=203)
31(7*31=217) 33(7*33=231) (7*5=35)35(7*35=245) 37(7*37=259) 39(7*39=273)
41(7*41=287)43(7*43=301)45(7*45=315) 47(7*47=329) (7*7=49)49(7*49=343)
51(7*51=357) 53(7*53=371) 55(7*55=385) 57(7*57=399) 59(7*59=413)
61(7*61=427) (7*9=63)63(7*63=441) 65(7*65=455) 67(7*67=469) 69(7*69=483)
71(7*71=497) 73(7*73=511) 75(7*75=525) (7*11=77)77(7*77=539) 79(7*79=553)
81(7*81=567) 83(7*83=581) 85(7*85=595) (7*29=87)87(7*87=609) 89(7*89=623)
(7*13=91)91(7*91=637) 93(7*93=651) 95(7*95=665) 97(7*97=679) 99(7*99=693)
101(7*101=707) 103(7*103=721) (7*15=105)105(7*105=735) 107(7*107=749)
109(7*109=763)
111(7*111=777) 113(7*113=791) 115(7*115=805) 117(7*117=819)
(7*17=119)119(7*119=833)
121(7*121=847) 123(7*123=861) 125(7*125=875) 127(7*127=889) 129(7*129=903)
131(7*131=917) (7*19=133)133(7*133=931) 135(7*135=945) 137(7*137=959)
139(7*139=973)
141(7*141=987) 143(7*143=1001) 145(7*145=1015) (7*21=147)147(7*147=1029)
149(7*149=1043)
```

D)

The first 120 numbers, respecting only the 9-Row:
$13(9 * 3=27) 5(9 * 5=45) 7(9 * 7=63) 9(9 * 9=81)$
$11(9 * 11=99) 13(9 * 13=117) 15(9 * 15=135) 17(9 * 17=153) 19(9 * 19=171)$
$21(9 * 21=189) 23(9 * 23=207) 25(9 * 25=225)(9 * 3=27) 27(9 * 27=243) 29(9 * 29=261)$
$31(9 * 31=279) 33(9 * 33=297) 35(9 * 35=315) 37(9 * 37=333) 39(9 * 39=351)$

```
41(9*41=369) 43(9*43=387) (9*5=45) 45(9*45=405) 47(9*47=423)49(9*49=441)
51(9*51=459) 53(9*53=477) 55(9*55=495) 57(9*57=513) 59(9*59=531)
61(9*61=549) (9*7=63)63(9*63=567) 65(9*65=585) 67(9*67=603) 69(9*69=621)
71(9*71=639) 73(9*73=657) 75(9*75=675) 77(9*77=693) 79(9*79=711)
(9*9=81)81(9*81=729) 83(9*83=747) 85(9*85=765) 87(9*87=783) 89(9*89=801)
91(9*91=819) 93(9*93=837) 95(9*95=855) 97(9*97=873) (9*11=99)99(9*99=891)
101(9*101=909) 103(9*103=927) 105(9*105=945) 107(9*107=963) 109(9*109=981)
111(9*111=999) 113(9*113=1017) 115(9*115=1035) (9*13=117)117(9*117=1053)
119(9*119=1071)
121(9*121=1089) 123(9*123=1107) 125(9*125=1125) 127(9*127=1143) 129(9*129=1161)
131(9*131=1179) 133(9*133=1197) (9*15=135)135(9*135=1215) 137(9*137=1233)
139(9*139=1251)
141(9*141=1269) 143(9*143=1287) 145(9*145=1305) 147(9*147=1323) 149(9*149=1341)
```

ad A) to D)

## A comment to the previous four tables:

The prime-end-digit " 1 " is dropped as a factor.
If it would be used as a factor, it would affect all numbers of the registry without changing their score, though the prime numbers would lose their status as such and become products (due to "1 * 3", "1 * 7", "1 * 9", "1 * 11", "1 * 13", etc).
So, the row starts with number 3, followed by $7,9,11,13,15,17,19,21$ and up to infinity.
The three tables (each until 119) show, how only the 3-Row, 7-Row and the 9-Row are already able to fill the registry with information for the numbers 3 to 119.
It contains no number, which doesn't provide sufficient information to clarify the here asked question.

## Conclusion:

As a side-effect, the factoring of "large numbers" (meaning products of prime-factors) may become redundant, given the registry is large enough.
The algorithm for this factoring, the "Castell-Fact-Algorithm", was already discovered by us in October 2019.
The uncapitalized multiplication on the right side of the numbers, concerns information for later numbers.

The uncapitalized multiplications on the left side of each non-prime number, concerns the factors, coming "from the left side" and having created the given product via multiplication.
These factors (if they are prime) have therefore created a "large number", which can be necessary for the RSA-Cryptography. At the same time though, they are the wanted prime numbers when it comes to factoring, which are here delivered along.

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