

Comparing the Use of Virtual Manipulatives and Physical Manipulatives in Equivalent Fraction Intervention Instruction

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Abstract

This paper describes a study designed to identify differences in learning trajectories related to the use of virtual and physical manipulatives during equivalent fraction intervention instruction. Changes in recent years in the approach to mathematical intervention have increased the need for research which identifies effective methods for the teaching of students with mathematical learning difficulties. Recommendations made from current intervention research emphasize the importance of students developing a proficiency in the use of representations through the use of manipulatives. Research indicates that virtual and physical manipulatives are effective tools of instruction. However, each manipulative type has its own unique affordances which affect the learning of specific concepts. The purpose of this study is to identify differences in students' learning trajectories related to the type of manipulative (virtual and physical) used during intervention instruction of equivalent fraction concepts.

Background

Due to the globalization of markets, advances in technology, and the overwhelming spread of information in today's society, mathematical skills have become increasingly important for success. Yet, there are a number of students who fail to acquire the needed mathematical skills during regular classroom instruction and the number of students needing special education services is increasing (Singapogu & Burg, 2009). As a solution, many programs, to more effectively support students with mathematical learning difficulties, have begun to shift from the traditional remediation approach to a response to intervention (RtI) approach. As an intervention approach, RtI focuses on the early identification and support of students who are struggling academically. The most common form of RtI intervention is a three tiered design. The first tier of intervention takes place in the regular classroom setting where all students receive research proven effective instruction. It is expected that at least 80% of the students will master the concepts taught during Tier I instruction (Fuchs, Compton, Fuchs, Bryant, & Davis, 2008). In Tier II intervention, students who did not achieve mastery in Tier I are given additional assistance. Tier II intervention is content specific and typically conducted by the classroom teacher in small group settings. Students who do not respond to Tier II intervention are considered to be "non responders" to intervention and will receive additional Tier III special education services (Fuchs et.al. 200).

Preliminary research results of RtI for mathematics intervention, although limited has been positive (Glover & DiPerna, 2007). Yet, developers of RtI programs have reported that the lack of available Tier II instructional materials and tools is limiting program implementation and research (e.g.: Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Fletcher, 2008; Gersten et.al., 2009; Glover & DiPerna, 2007). This study focuses on the use of virtual and physical manipulatives in equivalent fraction intervention instruction.

Representations

In a review of RtI literature, Gersten, et al.(2009) made eight research based recommendations for setting up effective RtI programs. The fifth recommendation reads:

Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas (p. 30).

Gersten, et al. (2009) explained that one of the most common difficulties experienced by students with mathematical learning difficulties is the ability to connect abstract symbols to

visual representations. They suggested that in regular classroom instruction, representations are not emphasized strong enough nor presented systematically enough to facilitate the scaffolding of learning for students with mathematical learning difficulties.

The purpose of using external representations (e. g. manipulatives, drawings, mathematical tables, etc.) in instruction is to aid students in their development of internal representations (Behr, Lesh, Post & Silber, 1983). Students' internal representations are: a) verbal /syntactic images, b) mental images, c) formal notation, and d) affective images including emotions, attitudes, beliefs and values (Goldin & Shteingold, 2001). As student's conceptual understanding of mathematical concepts develops, the power and flexibility of their internal representations grows. Students who have developed only partial internal systems of representations often experience difficulties in learning new concepts (Goldin & Shteingold, 2001). Physical and virtual manipulatives are tools of representation used to scaffold students' learning of mathematical concepts.

Physical and Virtual Manipulative Effectiveness

Physical manipulatives are concrete objects which students use to explore mathematical concepts through the students visual and tactile senses (McNeil & Jarvin, 2007). Virtual manipulatives are tools which are "interactive, web based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard & Spikell, 2002, p. 373). Advocacy for the use of manipulatives stem from the learning theories of Piaget, Bruner and Montessori that students develop and build knowledge as they move from concrete experiences to abstract thinking (McNeil & Jarvin, 2007). Piagetian theory suggests that children need to physically manipulate objects and then should be encouraged to reflect upon the meaning of the results of their physical actions (Baroody, 1989). These theories are supported by the research of studies such as the Rational Number Project, a project focusing on the development of representations through the use of manipulatives. Study results indicate that students using manipulatives significantly outperformed students taught using the more symbolic approach (Cramer & Post, 2002). The researchers described four ways manipulatives facilitated students development of fraction understanding: 1) students used the manipulatives to develop mental images of fraction meaning, 2) comparing manipulative objects helped students form correct methods for compare fractional sizes, 3) students use the manipulatives as references when justifying their answers, and 4) students using manipulatives developed less misconceptions.

The effectiveness of using physical manipulatives in mathematical instruction has been the focus of a large number of research studies. A review of the literature identified three meta-analysis reports. Suydam and Higgins (1977) reported that 11 of 23 studies reported finding significant differences in student achievement favoring the use of manipulatives, two studies favored not using manipulatives, and ten studies reported no significant differences between use and non use of manipulatives. Parham (1983) analyzed 64 studies and obtained 171 effect size scores comparing the use of manipulatives with non use on student achievement. Their analysis yielded an average mean effect size of 1.03, indicating a large effect size favoring manipulative use. Sowell (1989) reported, in their analysis of 60 studies that although manipulatives generally are found to be more effective than other types of instruction, but that there is a lot of variability in results, with some studies yielding large effect size scores while a few yielded negative effect size scores.

Moyer-Packenham, Westenskow and Salkind (2011) conducted a meta-analysis evaluating the effect of virtual manipulatives on student learning. The analysis of 70 effect scores obtained from 29 studies yielded a moderate average effect size of 0.37 when compared with the use of other methods of instruction. When virtual manipulatives were used alone as the primary tool of instruction and compared with instruction using physical manipulatives and with traditional classroom instruction, the averaged effect scores were a

small effect of 0.18 (33 effect scores) and moderate of 0.73(18 effect scores) respectively. Their analysis of the qualitative data suggested that virtual manipulatives have affordances of focused constraint, creative variation, simultaneous linking, efficient precision and motivation.

Students with Mathematical Learning Difficulties.

A search of the literature identified five studies investigating the effectiveness of instruction using physical manipulatives with students having mathematical learning difficulties. In two studies, Butler, Miller, Crehan, Babbit, and Pierce's (2003) and Witzel, Mercer, and Miller (2003), students with mild to moderate mathematical disabilities who participated in instruction using physical manipulatives scored significantly higher than students who did not use manipulatives. Similarly, results from three studies involving students with learning disorders' reported that achievement scores improved after students participated in instruction using manipulatives (Cass, Cates, Smith, & Jackson, 2003; Maccini & Hughes, 2000; Moch, 2001).

Four studies investigating the effectiveness of instruction using virtual manipulatives with students having mathematical learning difficulties were identified. Suh, Moyer, and Heo (2005) reported that when using virtual manipulatives, higher achieving students were more efficient and used more mental processes for finding answers while lower achieving students were more methodical in their use and more dependent on using the visual models when scaffolding between the pictorial and symbolic. Three studies reported positive effects when students receiving special education services used virtual manipulatives and two of the studies reported that the students using virtual manipulatives outperformed the students who did not use the manipulatives (Guevara, 2009; Hitchcock & Noonan, 2000; Suh & Moyer-Packenham, 2008). In summary, all of the identified studies in which manipulatives were used with students of differing abilities reported that students with mathematical learning difficulties benefitted from the use of manipulatives.

Combining Use of Virtual and Physical Manipulatives

Although limited, research indicates that there may be an advantage to combining the use of physical and virtual manipulatives in instruction. In a meta analysis, Moyer, Westenskow and Salkind (2011) identified 26 effect size cases of instruction combining the use of virtual and physical manipulatives. When instruction with combined use was compared with instruction using only virtual manipulatives, only physical manipulatives, and traditional classroom instruction, results produced a moderate averaged effect of 0.26 (9 effect size cases), a small effect of 0.20 (11 effect size cases), and a moderate averaged effect of 0.69 (6 effect size cases) respectively. These results indicate combining the use of virtual and physical manipulatives may be advantageous to student achievement.

Physical and virtual manipulatives have distinct affordances and disadvantages (i.e. many virtual manipulatives have explicit symbolic pictorial links, physical manipulatives involve tactile senses). Several researchers have reported that the affordances of each type of manipulative have resulted in variations of learning unique to the type of manipulative (Izydorczak 2003; Moyer, Niezgoda, & Stanley, 2005; Takahashi, 2002). As suggested by Behr et al. (1983) while a manipulative may be the most effective tool to use in teaching one concept, it may when used to teach a different concept impede student learning. They suggest more research is needed to identify which manipulative will best facilitate the learning of each concept. The purpose of this research study is to identify differences, related to manipulative type, in students' learning of the concepts of equivalent fractions. The research questions guiding the research are:

1. Are there variations in student achievement unique to the use of different instruction tools (virtual manipulatives, physical manipulatives or a combination of virtual and

physical manipulatives) in the learning of equivalent fraction concepts by students with mathematical learning difficulties?

2. Are there variations in the learning trajectories unique to the use of different instructional tools for intervention (virtual manipulatives, physical manipulatives or a combination of virtual and physical manipulatives) in the learning of equivalent fraction concepts by students with mathematical learning difficulties?

Methods

To answer these research questions, a study will be conducted providing preliminary Tier II intervention to fifth grade students who have not mastered equivalent fraction concepts in the regular classroom. The study uses a mixed methods approach of triangulating evidence from quantitative and qualitative data collected from pre/post/delayed post tests, session assessments, instructors' logs and session artifacts (task sheets, explore papers and video tapes). Data analysis will focus on the development of learning trajectories to be used as models of the progress made by students as they construct equivalent fraction understanding through the use of virtual and physical manipulatives. Not only will the overall effect of equivalent fraction learning be determined from the analysis of pre and post treatment data, but the researcher will also analyze data at the concept level, the individual lesson level, thus making it possible to identify effects of manipulative tool on the spectrum of students' development of equivalent fraction understanding.

Participants will be selected through a screening process. All students in the fifth grade classes of the participating schools will complete an equivalent fraction pre test. Students, except those participating in special education services for mathematics, scoring 70% or lower on the pretest, will be invited to participate in the intervention. Using an ability stratified method, students will be assigned to three groups, virtual manipulatives alone (VM), physical manipulatives alone (PM) or virtual and physical manipulatives combined (PM/VM). Intervention instruction groups will consist of two to four students. Participants will receive 5 days of equivalent fraction instruction, during which they will use their assigned treatment tool. Daily sessions will be approximately 45 minutes in length. The same lesson structure and content will be used for all three treatment groups, with minor adaptations made for tool differences. Each lesson will consist of five phases; pre-assessment, explore, apply, practice and lesson assessment. The lesson assessment consists of two parts; three lesson concept questions and eight cumulative fraction knowledge assessment questions. Lessons will be videotaped and all task sheets and student work will be collected for further analysis. At the conclusion of the intervention treatments students will complete a post test.

Analysis of the data will follow two paths: 1) analysis to identify differences in student achievement at the lesson, concept and summative levels, and 2) the development of learning trajectories showing knowledge and skill growth including the resolution of misconceptions and errors. The learning trajectories will be compared to identify differences related to manipulative type.

The following hypotheses are anticipated outcomes of this intervention study:

- Differences in the type of manipulative (physical or virtual) used creates differences in student's learning trajectories of equivalent fractions.
- Affordances of manipulative (physical and virtual) use are specific to each concept within the content domain of equivalent fractions.
- The type of manipulative (physical or virtual) used in instruction affects the occurrence and resolution of misconceptions and errors which are frequently experienced by students in their development of fraction understanding.

When developing intervention curriculum, teachers and curriculum developers must make important decisions about the type of instructional tools to be used. Although the research

literature indicates that both physical and virtual manipulatives are effective tools of instruction, research showing the affordances of each manipulative type for the learning of specific concepts is limited and does not answer the question of how to effectively combine the use of physical and virtual manipulatives in intervention instruction. The purpose of this study is to provide teachers and curriculum developers of intervention instruction, research that can be used in their selection of manipulatives in the different phases of students' equivalent fraction learning trajectories.

References

- Baroody, A. (1989). Manipulatives don't come with guarantees. *The Arithmetic Teaching* 37(2), 4-5.
- Behr, M., Lesh, R., Post, T., & Silver E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes*, (pp. 91-125). New York: Academic Press. Retrieved from http://www.cehd.umn.edu/rationalnumberproject/83_1.html
- Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction Instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, 18(2), 99-111.
- Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice*, 18(2), 112-120.
- Cramer, K. A., Post, T. R., & delMas, R. C. (2002). Students: A comparison of the effects of using commercial curricula with the effects of using the Rational Number Project curriculum. *Journal for Research in Mathematics* 33(2), 111-144.
- Fuchs, D., Compton, D.L., Fuchs, L.S., Bryant, J. & Davis, N.G. (2008). Making "secondary intervention" work in a three-tier responsiveness-to-intervention model: Findings from the first-grade longitudinal reading study of the National Research Center on Learning Disabilities. *Read Write* 21:413-436. DOI 10.1007/s11145-007-9083-9.
- Fuchs, L. S., Seethaler, P.M., Powell, S.R., Fuchs, D., Hamlett, C.L., & Fletcher, J.M. (2008). Effects of preventive tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. *Exceptional Children* 74(2), 155-173.
- Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J.R., & Witzel, B. (2009). *Assisting students struggling with mathematics; Response to intervention (RTI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://ies.ed.gov/ncee/wwc/publications/practiceguides>
- Glover, T. A., & DiPerna, J. C. (2007). Service delivery for response to intervention: Core components and directions for future research. *School Psychology Review* 36(4), 526-540.
- Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics, 2001 yearbook* (pp.1-23). Reston, VA: National Council of Teachers of Mathematics.
- Guevara, F. D. (2009). *Assistive technology as a cognitive developmental tool for students with learning disabilities using 2D and 3D computer objects*. (Master's thesis). Available from ProQuest Dissertations and Theses database. (UMI No. 1465252)
- Hitchcock, C. H., & Noonan, M. J. (2000). Computer-assisted instruction of early academic skills. *Topics in Early Childhood Special Education*, 20(3), 145-158.

- Izydorczak, A. E. (2003). *A study of virtual manipulatives for elementary mathematics* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3076492)
- Maccini, P. & Hughes, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children* 23(4), 465-489.
- McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulative debate. *Theory into Practice* 46(4), 309-316.
- Moch, P. (2001). Manipulatives work! *The Educational Forum* 66(1), 8.
- Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics*, 8(6), 372-377.
- Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of mathematical representations. In W. J. Masalski & P. C. Elliott (Eds.), *Technology-supported mathematics learning environments: Sixty-seventh yearbook* (pp. 17-34). Reston, VA: National Council of Teachers of Mathematics.
- Moyer-Packenham, Westenskow, Salkin (2011). Effects of virtual manipulatives on student achievement and mathematics learning. Manuscript in preparation.
- Parham, J. L. (1983). A meta-analysis of the use of manipulative materials and student achievement in elementary school mathematics (Doctoral Dissertation). Available from ProQuest Dissertations and Thesis database. (UMI No. 8312477)
- Singapogu, R. B. & Burg, T. C. (2009). Haptic virtual manipulatives for enhancing K-12 special education. Proceedings of the 47th Annual Southeast Regional ACM Conference Article No: 77.
- Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education*, 20(5), 498-505.
- Suh, J.M., & Moyer-Packenham, P.S. (2008) Scaffolding special needs students' learning of fraction equivalence using virtual manipulatives. In Proceedings of the International Group for the Psychology of Mathematics Education, pp. 4; 297-304. PME, 2008.
- Suh, J. M., Moyer, P. S., & Heo, H.-J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulative concept tutorials. *The Journal of Interactive Online Learning*, 3(4), 1-22.
- Suh, J. M., Moyer, P. S., & Heo, H.-J. (2005). Examining technology uses in the classroom: Developing fraction sense using virtual manipulative concept tutorials. *The Journal of Interactive Online Learning*, 3(4), 1-22.
- Suydam, M. N., & Higgins, J. L. (1977). *Activity-based learning in elementary school mathematics: Recommendations from research*. Columbus, OH: ERIC Center for Science, Mathematics & Environmental Education, Ohio State University.
- Takahashi, A. (2002). *Affordances of computer-based and physical geoboards in problem-solving activities in the middle grades* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3070452)
- Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121-131.