Probability in Mathematics: Facing Probability in Everyday Life

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Abstract
Understanding chance is essential to understand probability. The aim of this study was to find out how teenagers understand this idiom. We interview students from 6th, 8th and 11th grades. The results from the interviews pointed out that the idiom 'chance' is not clear enough. They understand better theoretical situations like throwing a dice. They understand the random and the certain in this situation. On the other hand, when they connect chance to everyday situations they relate to them as yes/no situations. They have difficulties in defining the random and the certain.

Introduction
Let's look at any typical "text book" activity devised for 8th grad students to understand what 'probability' is. The students are doing an experiment with tossing a coin (or a few coins together). They find that the ratio between the number of heads they got all together, and the number of coin tosses, is stabilized on 1/2. So they define the probability to get head on a random coin toss as this number. Do students, after this activity, understand what probability is? Are they able to apply this notion to everyday life? There is a need to find how teenagers understand probability. More specifically, how they understand 'chance' and what meaning do they apply when they use it in everyday life. These are the goals of this research.

Background
Piaget and Inhelder (Piaget and Inhelder, 1954) said that understanding probability means the ability to produce some quantification assessment (for the probability) that a certain event will occur. In order to give such assessment, one should understand both 'chance' and the usage of the combinatory operations. Piaget and Inhelder also said that understanding 'chance' is to understand total possibilities and randomness. Total possibilities events are events that will never occur (like throwing two dices and getting 14) or, events that one can be sure that they will occur (like throwing two dices and getting a result bigger then 0 and smaller then 13). They defined 3 stages for developing understanding of 'chance' and probability. Only at the third stage (from the age 11/12) one understands 'chance' and is able to use the combinatory operations.
According to the theory of Piaget and Inhelder, understanding 'chance' is essential for understanding probability. Understanding ‘chance’ is a combination of the discrimination between certain and uncertain events, and the understanding the distribution and the variety of the results.
Researchers in the last decade focused on understanding the variety of the results of an experiment that is repeated many times (like: Watson and Kelly, 2003; Watson et. al. 2003; Watson and Moritz, 2000, 2003; Sanches and Matinez, 2006). These researchers claim that understanding the variation of the results reflects actually understanding of the 'chance'. This understanding is combined of 3 aspects: understanding the randomness, understanding the structure of probability, and connecting this structure to empirical results.
The main aim of our research is to find what meaning teenagers, ranging from 11 years-old up to 17 y”o, give to the word 'chance' in everyday situations, including probabilistic situations.

Methodology

56 students (19 from 6th grade, 18 from 8th grade and 19 from 11th grade) were interviewed. In addition to free conversation, each interview included the following assignments:

1) Explain what is 'chance'.
2) Compose a sentence with the word 'chance'.
3) Explain what is 'almost zero chance'?
4) Read the story of the weather man. Did the weather man was wrong when he said that there is a 'almost zero chance' for the next day to be rainy?[1]
5) Playing "Ladders and Ropes" game. It is the same usual game with one additional rule: before throwing the regular dice, each player has to throw a special dice, which has 3 faces marked with a circle and 3 faces marked with a X sign. If the player gets the circle sign, he may proceed as usual and throw the regular dices. But if the player gets the X sign, the turn is passed over to the next player. Before the game, the students were asked what was the chance to get a 'circle' and what was the chance to get 'X'. When one player (or more) got many more Xs then circles (or vice-versa), the interviewer stopped the game and talked with the students about those chances again.

We asked about 'almost zero chance' in addition to 'chance', because we wanted to delve deeper into Piage's "understanding of the random and the certain". 'Almost zero chance' can easily be taken as the equation "chance = zero". While 'almost zero chance' **is fitted to** describe probability of random events, the notion of "chance = zero" is fitted to the probability of certain events. In the case of 'almost zero chance' there is still some probability that the event can occur, though it likely to be very poor.

Findings

Explanations to 'chance':

Most of the subjects said that chance means uncertainness. Like: "something that may happen", or: "a possibility that I'll get one out of two options". Very few subjects added some quantification. Like: "the percentages for something that will occur". (See table 1).

One note that almost 25% of the subjects could not explain what 'chance' is. They said that they understand what 'chance' is but they cannot explain it.

	6th grade		8th grade		11th grade	
	N	%	N	%	N	%
uncertainty	13	68	11	61	10	53
quantification	1	5	3	17	3	16
Could not explain	5	26	4	22	6	31
total	19	99	18	100	19	100

Table 1: Distribution explanation to 'chance'

Composing a sentence with 'chance':

All of the subjects composed a sentence with it the word 'chance', although there were some subjects that could not explain it. All the sentences were connected to everyday situations. In all the sentences, the meaning of 'chance' was uncertainty that an event will occur.

[1] Last night the weatherman said that there is almost zero chance of rain today. Today it did rain. Was the weatherman wrong?

One can classify the sentences into two categories. The first category - sentences with yes/no situations. Will the event happen or not. Like: “there is a chance that tomorrow we will win the football game”, or: “there’s no chance that I’ll eat tomatoes”. The other category - sentences with some quantification for 'chance'. Like: “there’s 20% chance that I’ll win the lottery game”, or: “low chances of having a rainy day tomorrow”. (See table 2.)

	6^{th} grade		8^{th} grade		11^{th} grade	
	N	%	N	%	N	%
Yes/no situation	12	63	11	61	11	58
quantification	7	37	7	39	8	42
total	19	100	18	100	19	100

Table 2: Distribution of composing a sentence with 'chance'

Note that almost 60% of the subjects use the word 'chance' for dichotomy situation (will this-and-that happen or not). There is a slight decline with age. The rest of the subjects (about 40%) added some quantification.

Explanations to 'almost zero chance':

All the subjects explained what is 'almost zero chance'. Most of the 6^{th} grade subjects (54%) said that it means “no chance at all”. Most of the 8^{th} grade subjects (56%) explained it well. The number of subjects from the 11^{th} grade who explained it well exceeded 68%.

The weatherman story:

More than 50% of the subjects from each grade said that the weatherman was mistaken. The rest said that he gave the right information, because their still was some probability that there will be rain the next day.

Consistency:

About 40% from the subjects at each grade showed right consistency. They said that 'almost zero chance' means a very small probability, and the weatherman was right. Most of the subjects from the 6^{th} grade (53%) showed wrong consistency. They said that 'almost zero chance' means no chance at all and the weatherman was wrong. Only 44% from the 8^{th} grade subjects showed the wrong consistency. The number decreased (32%) at the 11^{th} grade. There was also a third group - those who had no consistency. They said that 'almost zero chance' means no chance al all and that the weatherman was right, or the other way around - that 'almost zero chance' means a very small chance and the weatherman was mistaken. This group percentage increased from 11% al the 6^{th} and 8^{th} grades to 26% at the 11^{th} grade.
All the above details are gathered in table 3.

We can conclude that about 40% from all subjects understand correctly 'almost zero chance'. They understand that it means very poor chance, but there is still some chance that the event will occur. They understand it correctly when it is related to everyday situations. This rate does not change with age. However, about 40% from the subjects understand 'almost zero chance' as “no chance at all”, or the equation “probability = 0”. This rate decreases with age. (53% at the 6^{th} grate, 44% at the 8^{th} grade, 32% at the 11^{th} grade). An average of 16% from the subjects is inconsistent. This rate increases with age (from 10% at 6^{th} grade up to 26% at the 11^{th} grade).

		6th grade (N=19)		8th grade (N=18)		11th grade (N=19)		Total (N=56)	
		N	%	N	%	N	%	N	%
Explanation	chance=0	12	63	8	44	6	31	26	47
	very poor	7	37	10	56	13	68	30	54
The weather-man story	Wrong	10	53	9	50	11	58	30	54
	Right	8	42	9	50	8	42	26	47
consistency	wrong.	10	53	8	44	6	32	24	43
	right con.	7	37	8	44	8	42	23	41
	No con.	2	10	2	11	5	26	9	16

Table 3: Distribution of 'almost zero chance'

The 'Ladders and Ropes' game:

Before starting the game all member of each group of players were asked what is the chance to get a circle, and what is the chance to get an X. In each group there was at least one player who counted the number of circled and the number of X-s on the faces of the dice. All the subjects agreed that the chances are 50:50. This means they have the same chance to appear. The subjects then started playing the game, and the interviewer stopped the game when one symbol appeared many times more then the other. At this point the interviewer asked one or two players (those who got one symbol more then the other one) what symbol they gut most of the time? Then he asked them what are the chances to get a circle and to get an X, and if it is still 50/50?

Most of the subjects said that the chances to get a circle or an X did not change and they are still 50:50. Only few subjects (3 from the 6th grade, 2 from the 8th grade and 2 from the 11th grade) changed the probability to set a circle or an X according to the results they achieved. In one of the groups, one of the subjects said that, the chances are still the same 50:50 but he had “a special ability to throw the dice and get a circle”, which explained why he got so many circles.

At this point the interviewer asked them to explain how come the dice-tosses are so unbalanced if the chances are the same. The subjects from the 6th grade said that the differences between the numbers of the symbols are a matter of luck. The subjects from the 11th grade said that the numbers of all circles appeared at the game is very close to the number of the all X-s. Another answer was that in the short run one can get one symbol more times then the other, but in the long run the numbers will be almost equal.

Discussion:

From the findings we can see that most of the subjects understand chance as a word that is connected to uncertain events in the future. Following this one may think that they understand correctly what 'chance ' means. **But the key question is: How they understand uncertain events in the future**. That's why the subjects were asked to combine a sentence with 'chance'. We found that the majority of the subjects described yes/no events, and we also note that about a 1/4 of the subjects could not explain what 'chance' means (yet they could compose a sentence with the word). Had we found that the majority of subject who couldn’t explain it was of the youngest (6th) grade, we could have justified it by the limited ability of young students to express themselves. Yet, almost a 1/3 of the subjects from the 11th grade said that they know what 'chance' is and they cannot explain it. We believe this fact alone gives evidence to the fact that many students do not understand 'chance' at all, even though they often use it in everyday life.

Some connected ‘chance’ with yes/no situations, and if so they connect it to equal probability. For example:

Interviewer: Everything that may happen is “50% chance”?
Student: Yes.
Interviewer: Your friend told me that team so-and-so has no chance to win the next game. Do you agree with him?
Student: Yes, there is a great chance.
Interviewer: So there is a great chance. Is it still 50/50?
Student: Yes, it is 50/50.
Interviewer: Why?
Student: Because, if there is a chance - then it is 50/50.
Interviewer: What is the chance that tomorrow will be a nice day?
Student: Very high.
Interviewer: Very high, do you mean higher then 50?
Student: 50/50.
Interviewer: Still 50/50?
Student: Yes, because it may vary.

We got the same picture when we asked about 'almost zero chance'. From the subjects explanations one might conclude that they understand it well. But their reaction to the weatherman story showed that more then 50% of the subjects said that the weather prediction was wrong.
Almost everyone who explained 'almost zero chance' as ‘no chance’ also said that the weatherman was wrong. This was more common among the young subjects. Alternatively, most of the subjects who explain ‘almost zero chance’ as ‘a very poor probability’, said that the weatherman was right. This was more common among the older subjects. These two groups were almost the same in size – about 40% each.
We can see the same phenomena in 'chance' and in 'almost zero chance'. Theoretically, teenagers understand these notions well. But when students connect them to everyday life, they might change their meanings – these notions became synonyms to yes/no situations (will happen or will not happen).

The "Ladders and ropes" game demonstrated that most of the subjects showed a correct understanding of 'chance'. For example:

Interviewer: What did you get?
Student: Majority of X-s.
Interviewer: So what are the chances to get X?
Student: Still 50/50.
Interviewer: So how come you got so many X-s?
Student: Because when you throw the dice 3 times, and it does not tell anything (he threw the dice more then 3 times).
Interviewer: So how many times we need to throw the dice?
Student: about 100 times.
Interviewer: If I throw the dice 100 times then what?
Student: Most of the chances that you will get 50/50.

From this we can see that they understand that in the short run one might get one symbol more times then the other, but in the long run the numbers will be equals.

Earlier we claimed that understanding 'chance' in everyday situations is connected to yes/no situations. Here, the game reflects a well profound understanding. There is a contradiction. One way to settle the contradiction is to claim that only few of the subjects understood these ideas (the subjects were gathered in groups during the game). Even though the interviewer posed the question to one player, the other

answered, causing the first players, who were confused by the interviewer's questions, to adopt the answer and agree the student who answered. This is what Way and Ayres (2002) called the fragile probability knowledge and added that subjects do not feel the need to reason consistently.
Another way to explain the contradiction is that throwing a dice in a game like this presents a situation with many outcomes. Experiencing situations like this can build-up a correct understanding of probability. We have no doubt in our minds about the importance and contribution of a game like this to the understanding of probability. However, we believe that because of the many repetitions, and because students learned probability based on such games, students classify chance and probability as theoretic notions, and do not connect them to everyday life.

In this research, we found that there are two types of understanding 'chance'. One type is to understand 'chance' as a theoretical notion, and the other type is to understand 'chance' in connection to everyday life. Students display better understanding of probability when it is related to theoretical situations, especially older ones. When they are asked about everyday situations they connected them to yes/no situation and usually they estimate them as 50/50 chance. Amit and Jan (2006) pointed out that students are developing intuition to differ between numerical probability situations and empirical probability situations. Numerical probability situations are we here "theoretical situations", and empirical probability situations are everyday situations.

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