An Initial Examination of Effect Sizes for Virtual Manipulatives and Other Instructional Treatments

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Abstract
This paper is a meta-analysis comparing the use of virtual manipulatives with other instructional treatments. Comparisons were made using Cohen *d* effect size scores, scores which report treatment effect magnitude but are independent of sample size. Findings from 29 research reports yielded 79 effect size scores. Effect size scores were grouped and averaged to determine overall effects comparing use of virtual manipulatives alone, and in combination with physical manipulatives, to other instructional treatments. Results yielded moderate effects when virtual manipulatives were compared to all other instructional methods combined, large effects when compared to traditional instruction with textbooks only, and small effects when compared to instruction using physical manipulatives only. Combining physical and virtual manipulatives and comparing this treatment with other instructional methods resulted in moderate effect sizes for all comparisons.

Virtual Manipulatives and Mathematics Learning

Virtual manipulatives are "computer-based renditions of common mathematics manipulatives and tools" (Dorward, 2002, p.330). Moyer, Bolyard and Spikell (2002) define them as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Virtual objects such as pattern blocks, base -10 blocks, tangrams, and geoboards can be found as internet applets or as small stand-alone application programs. Many virtual manipulatives include features designed to focus the attention of learners by highlighting and enforcing mathematical concepts which support children's integrated-concrete knowledge (Dorward & Heal, 1999; Sarama & Clements, 2009). In the past two decades since the emergence of virtual manipulatives, there have been a number of research studies documenting the effects of virtual manipulatives as a mathematics instructional treatment. This meta-analysis synthesizes the research on this treatment by calculating averaged effects of virtual manipulatives on student achievement when compared with other instructional treatments.

Learning Mathematics with Virtual Manipulatives

Several constructs support the use of virtual manipulatives for learning mathematics concepts. *Representational fluency* is defined as a student's ability to transfer ideas easily from one representation to another, a skill which some researchers suggest can be strengthened through the use of technology by providing students with greater access to multiple and dynamic mathematical representations (Zbiek, Heid, Blume, & Dick, 2007). Virtual manipulatives develop representational fluency by linking symbolic, pictorial and concrete representations (e.g., placing a "90^{o}" beside a picture of a right angle); and by linking different types of representational models (e.g., a number line model showing ½ and a region model showing ½). By interacting with dynamic objects, students using virtual manipulatives learn to define, solve and prove mathematical problems by observing connections between their actions and the virtual objects (Durmus & Karakirik, 2006). When virtual manipulatives focus on linking representations, this can influence students' selection of problem solving methods. Manches, O'Malley and Benford (2010) observed that, during partitioning activities, students using virtual manipulatives used more compensation strategies while students using physical manipulatives used more commutative strategies.

Another construct which plays an important role in student learning is the *fidelity* of the technology tools (Zbiek et al., 2007). The degree of alignment between a tool and the mathematical properties is a measure of a tools' *mathematical fidelity*. The degree to which the tools reflect the users' thought processes is defined as *cognitive fidelity*. Seeing visually the consequences of their actions on virtual objects provides students with visual feedback as they test and prove new understandings. When using technology, cognitive fidelity can be even further enhanced as the user's actions are both represented and constrained, making the mathematical properties and relationships even more explicit for learners (Durmus & Karakirik, 2006). There are large collections of virtual manipulatives on the internet with resources linked to national mathematics standards (e. g., National Library of Virtual Manipulatives, http://nlvm.usu.edu; National Council of Teachers of Mathematics Illuminations http://illuminations.nctm.org; and Shodor Curriculum Materials http://shodor.com/curriculum/).

Research Questions

The purpose of this meta-analysis was to conduct an initial examination of effect sizes for virtual manipulatives when compared with other instructional treatments. Two research questions guided the analysis: 1) What are the effects of virtual manipulatives as an instructional treatment in mathematics on gains in student achievement? 2) What are the effects of virtual manipulatives as an instructional treatment in studies of differing durations?

Methods

The study used quantitative methods for a meta-analysis examining the effect sizes of multiple studies. Effect size scores were calculated and used to answer the research questions.

Data Sources

Following the search procedures and standard criteria outlined by Boote and Beile (2005), we conducted a comprehensive search of databases. These included electronic and manual library searches in educational and international databases such as ERIC, PsycInfo, Dissertation Abstracts, Web of Science, Google Scholar, and Social Sciences Index using search terms such as: virtual manipulatives, dynamic manipulatives, computer manipulatives, virtual tools, mathematics manipulatives, mathematics tools, technology tools, computer tools, mathematics applets, and computer applets. In addition to uncovering research on virtual manipulatives, the search located a large body of research focusing specifically on commercially developed dynamic geometry software (e.g., Geometer's Sketchpad, Cabri, and GeoGebra), which is a separate line of inquiry and beyond the scope of this analysis.

Criteria for Inclusion in the Meta-Analysis

From a collection of 135 publications discussing the use of virtual manipulatives, 74 articles and dissertations were identified as empirical research studies, 66 of which had been peer reviewed. The other 61 articles were papers expressing opinions, developing theories, or suggestions for instruction. To build a comprehensive base of studies, only three criteria were used to remove studies from the empirical pool originally identified. Sixteen studies were excluded because of study design, type of applet, and threats to validity such as history, mortality, instrumentation, testing, selection, regression, and maturation (Gall, Gall & Borg, 2003). A final total of 58 studies met our criteria for further review, and 29 of these studies contained effect sizes.

Analysis

Within the 29 studies, there were 79 effect score cases comparing virtual manipulatives with other instructional treatments. Effect size scores are used to report the magnitude of treatment effects and are independent of sample sizes, thus making the comparison of studies across multiple settings possible. Effect size scores were computed using gain scores, differences between post test scores, and F values used to calculate Cohen's d scores. The 79 effect score cases were grouped to obtain averaged effect size scores.

Results

The following results report comparisons between virtual manipulatives as an instructional treatment, a) with all other instructional treatments, b) with instruction using physical manipulatives only, and c) with traditional instruction using textbooks only. The effect sizes reported for each of the comparisons of the analysis are averages of 79 case effect sizes yielded from the 29 studies. Descriptions of effect sizes are based on the suggestions of Urdan (2010) that an effect size of less than 0.20 be considered small, effect sizes in the range of .25 to .75 are considered moderate, and those over .80 are considered large.

Effects of Virtual Manipulatives as an Instructional Treatment

The first research question focused on the effects of virtual manipulatives as an instructional treatment in mathematics on gains in student achievement. The following comparisons are presented in Table 1: a) all instruction using virtual manipulatives compared with all other methods of instruction; b) instruction in which only virtual manipulatives were used compared with all other methods of instruction, with instruction using physical manipulatives, and with traditional instruction using textbooks; and, c) instruction in which virtual manipulatives and physical manipulatives were combined as a treatment compared with all other methods of instruction, with virtual manipulatives used alone, with physical manipulatives used alone, and with traditional instruction using textbooks.

Table 1
Effect Size Scores for Virtual Manipulatives Compared with Other Treatments

Comparisons	Number of Comparisons	Effect Size
Virtual Manipulatives Used		
& Other Instructional Treatments	70	0.37 (0.44)*
Virtual Manipulatives Used Alone		
& Other Instructional Treatments (combined)	53	0.37 (0.46)*
Physical Manipulatives	35	0.18 (0.32)*
Traditional Instruction (textbook)	18	0.73
Virtual and Physical Manipulatives Used Together		
& Other Instructional Treatments (combined)	26	0.33
Virtual Manipulatives	9	0.26
Physical Manipulatives	11	0.20
Traditional Instruction (textbook)	6	0.69

Note: *Effect size with one outlier.

The comparison of all studies using virtual manipulatives for instruction with all other instructional treatments yielded a moderate averaged effect score (0.37; 0.44, with one outlier). The analysis of the 53 cases comparing instruction using only virtual manipulatives with other instructional treatments also yielded a moderate effect (0.37/0.46 with one outlier). An analysis of the 35 cases comparing instruction using only virtual manipulatives to instruction using physical manipulatives yielded a small/moderate effect (0.18/0.32 with one outlier); and the analysis of 18 cases comparing instruction using only virtual manipulatives to classroom instruction using textbooks yielded a moderate effect (0.73).

In the analysis of studies where virtual manipulatives were combined with physical manipulatives (VM/PM combined) for instruction and compared with other instructional methods, 26 cases yielded a moderate effect (0.33). In the comparison of VM/PM combined with the use of virtual manipulatives alone, nine scores yielded a moderate effect (0.26). VM/PM combined compared with physical manipulatives alone in 11 scores produced a small effect (0.20). Finally when VM/PM combined was compared with classroom instruction using textbooks this produced a moderate effect (0.69). In summary, the largest averaged effect scores for the virtual manipulatives were produced when comparisons were made between virtual manipulatives and classroom instruction using textbooks. Other comparisons produced moderate or small averaged effects. Overall the effect size results demonstrated that virtual manipulatives produced positive averaged effects on student achievement when they were used as an instructional treatment for mathematics teaching.

Effects of Virtual Manipulatives Based on Treatment Duration

The second research question focused on the effects of virtual manipulatives as an instructional treatment in studies of differing durations. This analysis examined the length of the instructional treatments when virtual manipulatives were used for instruction. Length of treatment categories were aggregated by days and number of effect size scores per category. The five categories for the analysis were 1 day, 2 days, 3-5 days, 6-10 days, and more than 10 days. These results are reported in Table 2.

Table 2
Effect Size Scores for Virtual Manipulatives by Length of Treatment

Length of Treatment	Number of Comparisons	Effect Sizes
1 day	10	0.13
2 days	5	0.36
3-5 days	12	0.21
6-10 days	10	0.48
More than 10 days	31	0.47 (0.62)*

Note: *Effect size with one outlier.

In approximately half of the comparisons students participated in instruction involving virtual manipulatives for durations longer than ten days. The shortest length of treatment (1 day) yielded the smallest averaged effect size score (0.13) when comparing instruction using virtual manipulatives to other methods of instruction. Treatments of 2, 6-10, and more than 10 days of virtual manipulative treatment, all yielded moderate average effect size scores (0.36, 0.48, and 0.47/0.62, respectively). The results of the comparisons indicate that studies of longer durations tend to report larger effect sizes while studies in which virtual manipulatives are used for shorter durations tend to report smaller effect sizes.

Discussion & Conclusions

The purpose of this study was to use a meta-analysis to synthesize the quantitative results from research on virtual manipulatives. From the meta-analysis, there are several patterns that emerged. Overall, the virtual manipulatives are an effective instructional treatment for teaching mathematics when compared with other instructional methods. It is also interesting to note that the average effect size scores for virtual manipulatives compared with traditional instruction using textbooks are larger than when comparing virtual manipulatives with physical manipulatives. However, combining virtual and physical manipulatives as a treatment compared with traditional instruction using textbooks resulted in some of the largest effects produced in this study. These results suggest that the virtual manipulatives have unique affordances that have a positive impact on student achievement in the learning of mathematics. The results also suggest that combining virtual and physical manipulatives for instruction provides students with representations available in each manipulative type that are a visual support for students and promote students' representational fluency.

Results of the meta-analysis also suggest that the length of treatment for virtual manipulatives influences the average effect size scores. This result is similar to other studies on instructional treatments showing that longer treatment durations provide more opportunity for the effects of a treatment to be determined through research. This result makes sense, particularly since there are various factors in the virtual manipulative environment which may be new to students, such as finding webpages or manipulating dynamic objects, and these activities take time for students to learn so that they can interact effectively with the virtual manipulatives.

Although averaged effect sizes indicate that virtual manipulatives are as effective, and may even be more effective tools of instruction than other methods, little is known about how learner characteristics, applet features or instructional methods affect student learning while using virtual manipulatives. Additional research is needed to determine if the use of virtual manipulatives as an instructional tool is more effective for some students than others. Although it has been suggested that virtual manipulatives could be successfully used in both gifted and intervention instruction, to date, there are limited research studies investigating differences in virtual manipulative use as related to student abilities. There is also great variability in applet features, structures and the amount of guidance they provide. To further enhance the use of virtual manipulative applets, research is needed which compares the effects of different applet characteristics on student learning and compares which applets are most effective for teaching which specific concepts within each mathematical domain. For example, Haistings (2009) indentified variations in learning when students used the same virtual manipulative applet with and without symbolic linking; and, Bolyard (2006) compared the effects of two different virtual manipulative applets for integer instruction. These types of investigations may help researchers to identify relationships between applet features and impacts on student learning and achievement.

This meta-analysis found that virtual manipulatives have a moderate average effect on student achievement when compared with other methods of instruction, and that larger effect size scores are produced when studies have longer treatment durations. While these results confirm the effectiveness of virtual manipulatives for mathematics instruction, they do not reveal *why* virtual manipulatives are effective. Further research on specific affordances that promote learning, effects for different mathematical domains, and implications of virtual manipulative use for different students will significantly contribute to our understanding of the features that make virtual manipulatives effective and will answer the question of why virtual manipulatives impact student achievement during mathematics instruction.

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