Exploring the challenges of teachers' and learners' understanding of solution strategies using whole numbers

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Abstract
This paper is a qualitative study and reports on work with both teachers and learners in nine schools in the Eastern Cape Province of South Africa on mathematical reasoning and problem solving, specifically related to developing computational fluency with whole numbers. As a facilitator who has been involved with the upgrading of mathematics in-service teachers, the study involved both teachers and learners, being able to calculate in multiple ways by using a variety of solution strategies in order to solve problems. Fluency require of both teachers and learners more than just memorizing a single procedure. It rests on an understanding of the meaning of the operations and their relationship with one another.
The work with learners and teachers involved them working individually on solving contextual word problems. The study showed that in most tasks, learners relied heavily on procedural understanding at the expense of conceptual understanding. Traditional standard algorithms appeared to have been learned in isolation from concepts, failing to relate them to understanding.

Introduction
The rationale for choosing whole number computation and studying the solution strategies of teachers and learners was triggered by the poor TIMSS results and my interest in the problem-centred approach that I had used in my own class.
The Third International Mathematics and Science Study (TIMSS, 1996) and the (TIMSS –R, 1998) had found that the performance of South African students was placed at the bottom of a list of over 41 countries that participated in the study. Having worked with many in-service teachers and learners over a number of years, trying to developing their computational fluency and reasoning skills, involving the four basic operations, I have come across a plethora of diverse solution strategies that both learners and teachers use in solving these problems.

As both studies from TIMSS showed no real difference in the performance of learners, a study on whole numbers was developed to ascertain whether learners and teachers were able to use different solution strategies when solving these problems. Encouraging learners to develop and use their own solution strategies is regarded as consistent with a move from being 'teacher-centred' to a more process driven problem solving 'learner-centred approach (Southwood & Spanneberg 1996). The Revised National Curriculum Statement (2002:1) reaffirms this when it states, "the outcomes encourage a learner-centred and activity-based approach to education." With the learner at the centre of the learning process, much more emphasis is placed on learners developing conceptual understanding and learning computational skills (Bransford, Brown & Cocking 1999).

Context
The Rhodes University Mathematics Education Project (RUMEP) is an independently funded non-governmental organisation (NGO), linked to the university, with the specific aim of improving the quality of teaching and learning mathematics in schools. The Project had its beginnings, on an informal, regional level, in 1983 and grew so rapidly in stature and effect

that it became a teacher development institute of the university in 1993, as a formal numeracy project.
A major aspect of the project is the upgrading of both primary and secondary mathematics teachers. These accredited courses represent a direct response to the challenge of reaching the many teachers in deep rural areas who have not had access to in-service training, in the past. The ACE programme is offered through two blocks of university based tuition over two years of part-time study. During these contact sessions, teachers are immersed into all five learning outcomes in Mathematics developing both their mathematics content knowledge and pedagogy.

Theoretical perspective

This study is underpinned by the social constructivist model (Vygotsky 1978) and the problem-centred approach. The social constructivist philosophy emphasises language, culture and the social milieu. In the social constructivist model of the teaching-learning process, four key elements interact and affect each other – the learner, the teacher, the task and the context. As knowledge is socially constructed, the classroom is seen as an extension of the learners' environment. That knowledge which the learner knows is built on the existing knowledge gained through social interactions other than those found in the formal classroom.
Problem solving is consistent with the constructivist philosophy and my submission that learners are encouraged to invent their own procedures as advocated by (McClain & Cobb 2001) so that learners build their own meaning for themselves in order to better understand the concepts and skills of mathematics is pertinent here.

Methodology

The study is qualitative in nature and lies in the interpretive paradigm. It deals with individuals and is interested in describing processes rather than just an outcome or end result (Cohen & Manion 1994; Mwira & Wamahui 1995). Grounded theory was the underpinning methodology selected for the study. It was postulated by Glaser & Strauss (1967) and appropriate for the study as it was a small scale investigation into the solution strategies of learners. The initial justification for this research method was that I intended to develop categories of children's solution strategies. However, when examining the strategies, I found that the level of each task only allowed me to go as far as the first type of coding, namely open coding.
To ascertain the problem solving ability of learners and teachers and their ability to use different solution strategies, nine schools (both teachers and learners) in the Northern region of the Eastern Cape Province took part in the study.

The research instruments consisted of a test, individual clinical interviews with each of the nine learners and teachers and a structured teacher interview schedule.

The majority of the learners were isiXhosa speaking. However for transparency sake, all the questions were translated from English into Afrikaans and isiXhosa. The test consisted of 15 multiple choice type questions and 12 problem type word problems. Grade 7 learners were chosen because it was felt that learners at this age are able to articulate their thought processes and their communication skills are also sufficiently developed at this level.

My interest in the test was to find out how the learners and teachers confronted the problems and what strategies they had used which made sense to them. Further I was interested to see whether the solution strategies chosen, were the same or different from those their teachers had used.

The individual interview schedule was semi-structured in nature (Bogdan & Biklen 1992) and contained a checklist of suggested questions. However, not all questions were predetermined. This kind of interview allowed me to have more flexibility and freedom to explore the solution strategies adopted by the participants and whether they could explain carefully what they had done. The goal and emphasis of using the semi-structured interview was to probe for understanding.

A short, additional structured interview schedule was drawn up asking participant teachers to comment on aspects of the test. The intention behind doing this was to gauge whether the tasks were too difficult for their classes and whether translating the tasks into their mother tongue had any effect on how learners approached them.

Interpretation and discussion of solution strategies

The TIMSS Curriculum Framework was used in this study to place each task into categories and into performance expectations. The performance expectation component refers to the cognitive dimension and describes the kinds of performance or behaviours that might be expected of learners. I shall discuss three whole number problems from the test.

Task 1

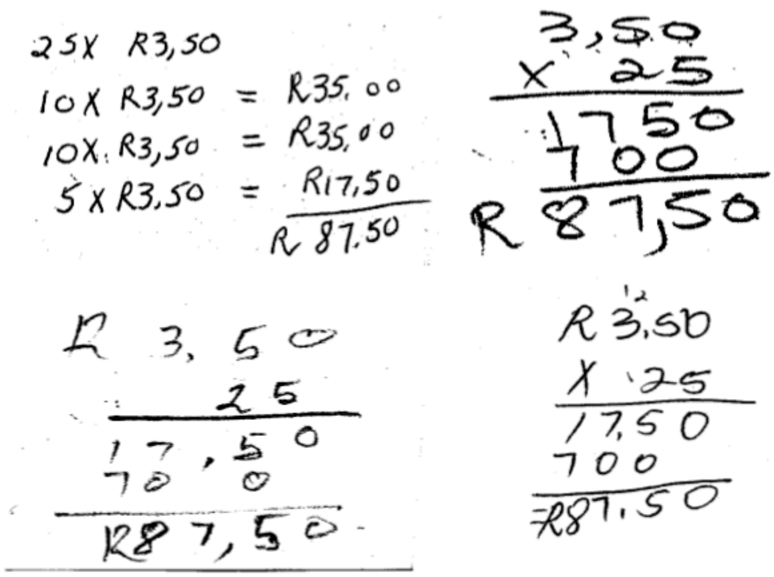
25 learners go on an outing to the beach. They each buy an ice-cream which costs R3, 50.
How much must they pay altogether?
Open coding was used and the following strategies were developed.

Learner	**CS**	**IS**	**Strategies**	**Teacher**	**CS**	**IS**	**Strategies**
1	√		Vertical algorithm	1	√		Vertical algorithm
2		×	Counting strategy	2	√		Decomposition of the multiplier
3		×	Vertical algorithm	3	√		Vertical algorithm
4	√		Mathematical model	4	√		Decomposition of the multiplier
5	√		Vertical algorithm	5	√		Vertical algorithm
6		×	Vertical algorithm	6	√		Fraction multiplication
7		×	Fraction multiplication	7	√		Decomposes multiplier
8		×	Counting strategy	8	√		Vertical algorithm
9	√		Vertical algorithm	9	√		Short method of X

Content area category	Number
Subcategory	Decimal Fractions
Subordinate subcategory	Properties of operations
Performance expectation	Using routine procedures
Subcategory	Performing routine procedures

Only 44% of the learners managed to solve this problem with the majority using the traditional vertical algorithm strategy. Only a few were able to explain the process involved. Many got lost along the way because they lacked the mastery of the multiplications tables. Strategies identified were: decomposing the multiplier, using the vertical algorithm, use of a mathematical model, a counting strategy and fraction multiplication. This example was correctly answered by all nine teachers (100%).

Teacher and learner strategies

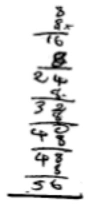
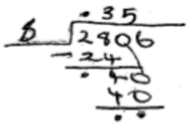
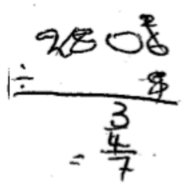
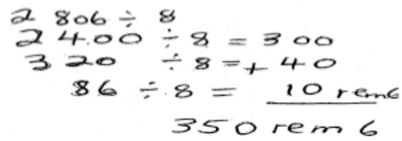


Task 2
Farmer Zodwa and his workers pick apples. They pick 2 806 apples and place them in packets with 8 apples to a packet. How many packets do they fill? Are there any apples left?

Learner	**CS**	**IS**	**Strategies**	**Teacher**	**CS**	**IS**	**Strategies**
1		×	Vertical algorithm	1		×	Vertical algorithm
2		×	Unclear strategy	2		×	Partitioned dividend
3		×	Unclear strategy	3	√		Partitioned dividend
4		×	Factors of 2 and 4	4	√		Partitioned dividend
5		×	Partitioned dividend	5		×	Vertical algorithm
6		×	Vertical algorithm	6		×	Vertical algorithm
7		×	Partitioned dividend	7		×	Partitioned dividend
8		×	Guess work	8		×	Partitioned dividend
9	√		Vertical algorithm	9	√		Vertical algorithm

Only 11% of the learners solved this problem while only 33% of the teachers accurately answered it. Most learners relied on memorisation but got stuck halfway as they tried to solve the problem. Teachers still teach division as a 'goes into' model which we know is an ineffective model as digits are treated separately (Fosnot & Dolk 2001). Teachers tend not to emphasis the reciprocal relationship between multiplication and division or what the meaning of a remainder is.

Learner and teacher strategies

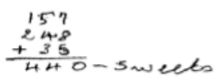
Task 3

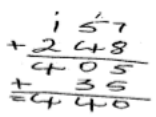
Mrs Khumalo has a bag of sweets to give to her Grade 7 classes. She gives the first class 167 sweets and the second class 248 sweets. She then has 35 sweets left in her bag. How many sweets were in her bag at the start?

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	√		Vertical algorithm	1	√		Vertical algorithm
2	√		Decomposition of numbers	2	√		Vertical algorithm
3	√		Vertical algorithm	3	√		Vertical algorithm
4	√		Vertical algorithm	4	√		Vertical algorithm
5	√		Vertical algorithm	5	√		Vertical algorithm
6	√		Vertical algorithm	6	√		Horizontal and vertical algorithm
7	√		Vertical algorithm	7	√		As above
8	√		Vertical algorithm	8	√		Vertical algorithm
9	√		Vertical algorithm	9	√		Vertical algorithm

There was a 100% success rate with this problem. This shows that some teachers spend more time teaching this operation at the expense of the other operations.

Learners' strategies





Findings

Most of the solution strategies that the learners used were straight forward procedures that they had learnt. They relied mostly on procedural understanding at the expense of conceptual understanding. The solution strategies of whole numbers adopted by the learners in the study were similar to the whole number solution strategies used by their teachers. A few teachers and learners did employ their own constructed solution strategies. They were able to make sense of the problems and to 'mathematize.' Language played a role in that learners sometimes struggled to communicate their thought processes in a coherent manner, even though the problems had been translated into mother tongue as well.

Challenges

Teachers should ensure that learners be given sufficient opportunities to solve problem-solving type word problems. Besides addition, the other whole number operations must also be given the recognition they deserve. In each lesson planned, time should be given to allow learners to master basic skills to enable them to use this information when planning their solution strategies. The need to take cognisance of language problems is a further aspect that teachers need to take into consideration. They should note that if word problems are written in English and mother tongue, reading, comprehension and encoding errors would be lessened. To make sense of the mathematics, it is important for English second or third language speaking learners to be allowed to rely on their own language, since background knowledge is the basis of any learning process in mathematics. The implication is that teachers need to spend time allowing learners to look at the processes involved in arriving at a solution, rather than just focusing on the solution. By allowing learners to develop their own solution strategies, teachers will be sensitized to the thinking and reasoning of learners as they strive to make sense of the mathematics. Teaching algorithms as fixed procedures restricts the thinking ability of learners to reason, communicate and consequently, their ability to do mathematics. Teaching for understanding should be emphasized at the expense of 'teacher taught procedures'. The idea of being computationally fluent will result in learners being able to explain and analyse their methods and this will result in their developing efficient, accurate and flexible strategies. As their knowledge of different strategies grows, so does their computational fluency.

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