Workshop: Error Analysis of Mathematics Test Items

Rencia Lourens; Nico Molefe and Karin Brodie

Florencia.Lourens@wits.ac.za; Nicholas.Molefe@wits.ac.za; Karin.Brodie@wits.ac.za

School of Education, University of the Witwatersrand, Johannesburg

The workshop will help mathematics teachers develop an understanding of learner errors and misconceptions and how these errors and misconceptions can be embraced in the teaching and learning of mathematics. The workshop will draw on an activity developed by the Data Informed Practice Improvement Project (DIPIP). Some examples of learner errors from an international test will be given. Participants will be given the opportunity to discuss and identify the mathematical content needed to answer the question, analyse the correct answer and then analyse the incorrect answers, identifying the reasons for learner errors and the misconceptions that produce these errors.

Background:

The Data Informed Practice Improvement Project (DIPIP) is an innovative professional development project aiming to develop sustainable professional learning communities amongst mathematics teachers. Teachers engage with data from their classrooms and work towards understanding learners' errors better. This is done by trying to understand why learners are making errors and how teachers should respond to and work with the errors.

We have developed a number of activities in the project to engage teachers with their learners' thinking. One of these activities is error analysis, where we look at test items and have conversations about the errors that learners make, and the reasons behind these errors. We work with teachers to support them in understanding and engaging with learner errors in the classroom, and to view errors as reasonable (Nesher, 1987; Smith, diSessa & Roschelle, 1993; Drews, 2005).

Purpose of the workshop:

To share ideas on how teachers can be assisted in working with errors in the teaching and learning of mathematics by

- Engaging with learner errors,
- Making meaning of the learners' thinking behind the errors
- Recognising that errors can show valid learner thinking.

Theoretical ideas on error analysis:

Error analysis forms part of the key activities in the DIPIP project. Errors are a result of a "consistent conceptual framework based on earlier acquired knowledge", called misconceptions (Nesher, 1987: 33) and make sense to learners in terms of their current thinking. Learners do not just make errors - these errors make sense to them as a result of the conceptual links that they make to knowledge they acquired previously. It should be noted that misconceptions may lead to correct answers, even when the mathematical thinking producing these answers is partly or entirely incorrect (Nesher, 1987). A classic example of this is the case of Benny as discussed in Erlwanger (1973).

Borasi (1987) views errors as "springboards for enquiry". In other words, errors can raise important issues for further exploration of the mathematics. Because errors make sense to those who make them, it's important that errors be embraced in the teaching and learning of mathematics, and not be ignored, or merely corrected. It is also important to know how errors and misconceptions can inform our instructional practices. Errors "provide evidence that the expected result has not been reached, and that something else has to be done" (Borasi, 1987: 4) and errors can therefore be used to "investigate the nature of fundamental mathematical notions" (Borasi, 1987: 5). While we agree with Borasi (1987) that errors can be springboards for enquiry we need to acknowledge that errors can be an indication to teachers of misconceptions that exist and need to be addressed. Teachers would normally not teach errors or misconceptions. For learners to develop misconceptions is a normal part of learning. All learners develop misconceptions at some point, even those who have good teaching. When teachers teach in ways that do not embrace errors, there is a likelihood that misconceptions might develop or be perpetuated. It is for this reason that teachers need to understand the type of errors that learners make and also understand why they make those errors. Understanding learners' errors can help teachers develop teaching strategies that can engage with learners' misconceptions. Teachers can never entirely prevent errors, so it is important that they can deal with them as they come up.

Assessment tasks on which the workshop is based:

The workshop activity is based on results of a standardised, international test that learners of participating schools have written, with focus on Algebra. This test was developed as part of the research programme "Concepts in Secondary Mathematics and Science" (CSMS) (Hart, 1981), and has been extended by "Increasing Competence and Confidence in Algebra and Multiplicative Structures" (ICCAMS). According to Hart (1981), there are six different ways of interpreting and using letters: letter evaluated, letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalised number and letter used as a variable (p.4). Hart (1981) discusses these categories as follows:

Letter Evaluated – when the response suggests that the letter is given a numerical value instead of being treated as an unknown or generalised number.

Letter Not Used – letter is acknowledged without being given a meaning or it is simply ignored

Letter as Object – letter used to denote an object

Letter as Specific Unknown – letter thought of as a particular but unknown number

Letter as Generalised Number – letter seen as being able to take several values

Letter as Variable – letter seen as representing a range of values, which is more of a dynamic view

The above categories are used as guidelines for analysis of errors. When teachers use these categories to analyse learner responses, they will identify the type of errors that learners have made and discuss these errors in relation to the teaching and learning of mathematics. Templates that we have developed through DIPIP are used to analyse errors that learners make.

Items chosen:

For this workshop we have chosen six items to look at and will use the answers of different learners to work on the error analysis activity with participants. We will identify the errors that learners made and workshop participants will discuss why we think learners made those errors. The items were chosen because of the richness in the errors that the learners made.

The workshop will provide participants with opportunities to look at classroom data in different ways. Data from classroom can help teachers develop methods of looking deeply into learner needs and see how these learner needs can inform teachers' learning needs (Earl & Katz, 2006)

Boudett, City, & Murnane (2006) argue that teachers can deepen their understanding of learners' strengths and misconceptions by looking collaboratively at their learners' work and talking about it in meaningful ways. In looking at their learners' work teachers can get opportunities to understand their learners' thinking, especially when learners make errors.

Analyses:

In analysing the items participants in the workshop will use a template that will be given to them to look at the following:

- Test Item

  Here we will look at the question and provide participants the opportunity to answer the question in as many different ways as possible.

- Content Description

  Participants will need to identify the mathematical content needed to answer the question.

- Analyses of Correct Response

  Analysing what knowledge, skills and procedures are required to get the correct response for this question

- Analysis of Errors

  This is the stage in the analysis that normally brings about the most discussion. Looking at the answers of the learners we look at the actual errors made by learners. Not all errors are discussed in depth, but the ones which are common. The aim is to understand the thinking processes of the learners and to identify any misconceptions that could have contributed to this particular error. All possible reasons for the error should be discussed.

- Critical Concept(s)

  The final stage, having done the error analysis, is to identify the critical concept(s) needed by the learner. The questions asked are what is missing between the error and the correct response. This can be seen as the link between the error and the correct response. These critical concepts become the learning needs of the learners and in the DIPIP project lessons would be developed around this concept.

Conclusion:

It is important for teachers to understand the role that errors play in the teaching and learning of mathematics. The workshop will try and raise an awareness of how errors can be embraced in mathematics classrooms instead of being ignored, or merely corrected. The error analysis activity can provide participants with skills to handle errors that learners make in class as well as the errors they make when they write their homework.

We will provide participants with an opportunity at the end of the workshop to give an evaluation of how the workshop went; what they gained; what works well for them and what does not; and also make input on any matter they deem necessary in working with learner errors and misconceptions.

References:

Borassi, R. (1989), "Exploring Mathematics through the Analysis of Errors" in *For the Learning of Mathematics 7*(3), pp. 2-8

Boudett, K. P., City, E. A., & Murnane, R. J. (2006). *Data Wise: A step-by-step Guide to Using Assessment Results to Improve Teaching and Learning*. Massachusetts: Harvard Education Press.

Drews, D. (2005). "Children's mathematical errors and misconceptions: perspectives on the teacher's role" in A. Hansen (Ed.), *Children's Errors in Mathematics*

Earl, L. M., & Katz, S. (2006). *Leading Schools in a Data-Rich World: Harnessing Data for School Improvement*. California: Corwin Press

Erlwanger, S. (1973). "Benny's misconceptions of rules and answers in IPI mathematics", *Journal of Children's Mathematics Behaviour,* 1(3), pp. 157 - 283

Hart, K.M., (ed.) 1981, *"Children's understanding of Mathematics: 11 – 16*", John Murray Publishers, London, pp. 102 – 119.

Nesher, P. (1987). Towards an Instructional Theory: the Role of Student's Misconceptions. *For the Learning of Mathematics, 7*(3),), pp. 33-39.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions Reconceived: A Constructivist Analysis of Knowledge in Transition. *The Journal of the Learning Sciences, 3*(2),), pp. 115-163.