MATHEMATICAL COMPETENCE ASSESMENT OF A LARGE GROUPS OF STUDENTS IN A DISTANCE EDUCATION SYSTEM

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Abstract

We present a model for the assessment of mathematical competence of a large group of students in a distance education system. The model can be applied automatically and allows not only a summative evaluation type, but it has also educational character. We present some results of applying the model to a real situation for the Course *Applied Mathematics to Social Sciences* taught at UNED of Spain as a part of the Foundation Course for access to University for people 25 years old and over.

Mathematical Competence

The use of the term competence has set out as the cornerstone for the design of teaching and learning process in diverse areas and educational levels. Currently, in Spanish universities, all degrees must be designed in accordance with the teaching and learning model aimed to developing learning skills. It is not possible here to make a profound discussion on the various meanings of the term. For purposes of this paper we understand that "*competence is values, attitudes and motivations, as well as knowledge, skills and abilities, all as part of a person who is insert in a given context, considering also that he/she learns continuously and progressively throughout life*" (Sevillano, 2009).

According to the above idea, to define the mathematical competence it is necessary to identify the components that comprise it. That is, mathematical competence is developed when successfully integrated mathematical knowledge and skills, so they emerged in different areas of personal performance directed by certain values and attitudes. Ramos, 2009, includes an extensive discussion on how to achieve the mathematical competence. The determination of the components leads to the following specifications, Ramos *et al.*, 2010:

- *Mathematical knowledge* formed around certain key ideas that have historically set the scope of mathematics and the core of mathematical thinking. These ideas include: i) *Mathematical language*, ii) *Quantity*, iii) *Space and shape*, iv) *Change and relationships*, v) *Uncertainty*. Each of these sections includes a set of concepts that formalize the different experiences of men in real situations that present aspects of mathematical nature. It is clear that among those sections there are many common points and any attempt to strictly separate them leads only to artificial situations. However, the above classification is useful not only for its traditional character, but also for its important methodological implications. The description and contents of the five sections can be found in the text by Hernández, Ramos, Velez and Yáñez, 2008.

- *Mathematical capabilities* that are required at different stages of the mathematical process, putting into action the mathematical knowledge in a given context. These capabilities, according to PISA, 2003, 2006, 2009, can be stated as: i) *Thinking and reasoning*, ii) *Argumentation,* iii) *Communication* iv) *Modelling*, v) *Problem posing and solving*, vi) *Representation*. vii) *Using symbolic, formal and technical language and operations,* viii) *Use of aids and tools*. The cognitive activities that include these capabilities are three levels of development or skill clusters: *reproduction* cluster, the *connections* cluster and the *reflection* cluster, each of which represents, respectively, a higher level of development of mathematical competence. It is also possible to consider the following contexts*: personal, educational/occupational, public and scientific*. The meaning and scope of the above expressions is conveniently detailed in PISA 2003, 2006, 2009, (*vid.* Ramos 2009, Ramos *et al*. 2010).
- *Attitudes* that show: i) *Security and confidence with information and situations that contain mathematical elements*, ii) *Respect and enthusiasm for certainty through the correct reasoning*.

Evaluation of mathematical competence

One of the key issues of a teaching and learning model designed to develop skills is how to truly evaluate the achievement of the desired competences. Traditional models aimed at acquiring knowledge include an evaluation system designed to observe mainly whether or not a student has certain knowledge. In a model of competences we must go further and try to assess whether or not students have acquired the expected competences. In the assessment of competences we should keep in mind that these, in addition to knowledge, include skills, abilities, attitudes and values that the student has to develop. We then face the problem of assessing the level a student show in certain qualities, that can only be seen with a degree of subjectivity and are more difficult to detect by the usual assessment methods.

Generally, the competence models use assessment systems such as oral presentations, analysis of data or texts, skills practice while the individual is under observation, professional folder (portfolio), reports field work, written thesis, or similar. However, these activities encounter practical problems that can make them inapplicable in certain situations. In particular, we note that they are usually only practicable in groups of few students, they represent a major increase in workload for both students and faculty and are difficult to apply in some contexts such as distance education model. If we do not want to refuse to use a competence model, the alternative is to investigate the possibility of adapting the traditional assessment schemes so that they can be used as an acceptable method for assessing competence.

Ramos *et al*., 2010, present the theoretical ideas for the development of a competence evaluation system inspired by traditional systems, assessing not only knowledge but also skills and attitudes. The idea is to design assessment activities provided with a number of appropriate characteristics so that it is possible to assess the degree of acquisition of the various components and subcomponents of the competences.

The practical application of this theoretical model of evaluation to the case of the mathematical competence, in a large group of students and in a distance education system, leads to designing evaluation activities with certain attributes that allow the combination of simplicity of the traditional evaluation with the objectives of the competency assessment. We use automatic evaluation tests in order to obtain an objective qualification within a reasonable period of time. Moreover, the design of these tests include a quantification of all aspects that make up the components and subcomponents of the competences, that is, the acquisition of knowledge, skills and attitudes within a given context. Finally, it is necessary to consider an indicator for the accuracy or adequacy of the response to a particular reference. The above considerations lead us to define a test or evaluation activity as an object identified by the attributes that are listed in Table 1. Specific examples of such tests can be seen in Ramos *et. al.*, 2010

TEXT	ASSESSMENT			CORRECTION INDICATOR
	Knowledge	Capabilities	Attitudes	
Statement (TST). Correct alternative (TCA) Distractor 1 (TD1) Distractor 2 (TD2)	Mathematical Language (VKL) Quantity (VKQ). Space and Shape (VKS) Change and Relationships (VKC) Uncertainty (VKU)	Thinking and reasoning (VCT) Argumentation (VCA) Communication (VCC) Modelling (VCM) Problem posing and solving (VCP) Representation (VCR) Using symbolic, formal and technical language and operations (VCST) Use of aids and tools (VCU)	Security and Confidence (VAS) Enthusiasm for certainty and correct reasoning (VAE)	Success. (S) Failure (F) No answer (W)

Table 1

Thus, mathematical competence can be assessed by applying an evaluation form consisting of a number N of activities with the characteristics in Table 1. To do this, we must have a function of the attributes that, based on indicators of correctness of each of the N tests, provide an overall score, either quantitative or qualitative, in each of the components and subcomponents of the competences.

The quality of an evaluation form can be assessed by considering the overall characteristics of the attributes of the activities that it comprises, like the mean, minimum, maximum, range, etc. of the valuation of knowledge and skills. Thus, it is possible to compare different forms and even prepare forms that meet certain requirements desired by the teacher.

Case study

We present in this section the results of applying in practice the model described above. The framework is the subject *Mathematics Applied to Social Sciences* in the exam of June 2010. The examination form consists of ten evaluation questions. We use a numerical scale from 0 to 4 to assess the intensity with which a question measures the level of acquisition of each of the subcomponents of the competences. The attributes of each question in the form are summarized in Table 2. The indicator correction is the same for all question: S = 1, F =- 0.25 and W = 0.

Question	Knowledge					Capabilities								Attitudes	
	VKL	VKQ	VKS	VKC	VKU	VCT	VCA	VCC	VCM	VCP	VCR	VCS	VCU	VAS	VAE
1	1	2	1	4	0	2	1	1	1	1	2	3	1	1	0
2	1	4	0	0	0	1	0	2	1	2	2	3	3	1	0
3	4	2	0	0	0	4	1	2	3	2	2	2	2	1	3
4	0	0	0	0	4	4	2	2	4	2	1	1	0	4	1
5	2	2	4	1	0	2	2	2	3	2	3	2	2	1	1
6	2	1	1	4	0	2	2	2	3	3	2	3	2	1	1
7	1	0	4	0	0	3	3	3	3	2	3	2	0	3	0
8	4	0	0	0	0	3	4	3	1	1	1	2	0	1	4
9	0	3	0	0	4	3	2	3	4	3	4	2	4	4	1
10	0	4	0	0	0	1	1	1	2	2	2	2	4	1	1

Table 2

The student responses are collected by automatic reading sheets and form a vector $R = (R_1, R_2, K, R_{10})$, where, R_i, $i=1,...,10$, takes one of the values *TCA(i), TD1(i), TD2(i)* or it is a blank or null answer. If we denote

$$E_i = \begin{cases} 1 & \text{if } R_i = TCA(i) \\ -0.25 & \text{if } R_i = TD1(i) \text{ or } R_i = TD2(i) \\ 0 & \text{otherwise} \end{cases}$$

then the traditional numerical grade, on a scale of 0 to 10, is calculated by the formula $E = \text{Máx}\left\{0, \sum_{i=1}^{10} E_i\right\}$.

For an independent assessment of each of the components of the competences, the above expression can be extended in several ways. For instance, it is possible to assign a different weight to each response, according to different attributes of the question. However, the current assessment model considered only assesses the different subcomponents of competence, using the data included in Table 2. Thus, the score obtained in knowledge, K_j, *j=1,...5*, capability C_k, *k=1,...,8*, and attitude A_l, *l=1,2* can be expressed as

$$K_j = \text{Max}\left\{0, \frac{10}{k_{.j}} \sum_{i=1}^{10} k_{ij} E_i\right\} \quad C_k = \text{Max}\left\{0, \frac{10}{c_{.k}} \sum_{i=1}^{10} c_{ik} E_i\right\} \quad A_l = \text{Max}\left\{0, \frac{10}{a_{.l}} \sum_{i=1}^{10} a_{il} E_i\right\}$$

where k_{ij} is the valuation of question *i* in knowledge *j*, c_{ik} is the valuation of question *i* in capability *k* and a_{il} is the valuation of question *i* in attitude *l*, given by Table 2; moreover, $k_{.j} = \sum_{i=1}^{10} k_{ij}$, $c_{.k} = \sum_{i=1}^{10} c_{ij}$ and $a_{.l} = \sum_{i=1}^{10} a_{il}$ are, respectively, the total of the values of the knowledge (*j=1,...,5*), capabilities (*k=1,...,8*), and attitudes (*l=1,2*) in the form. Finally, the average values can be calculated:

$$K = \sum_{j=1}^{5} \frac{K_j}{5}; \qquad C = \sum_{k=1}^{8} \frac{C_j}{8}; \qquad A = \sum_{l=1}^{2} \frac{A_l}{2}; \qquad EC = \frac{K + C + A}{3}$$

providing a measure of the level of each component of competence, (*Knowledge K*, *Capabilities C*, *Attitudes A*) and overall assessment of the mathematical competence *EC*.

Table 3 shows some records in the database of results. Each row corresponds to a particular student, while the columns include identification (ID), the valuation of each of the five subcomponents of knowledge, the eight subcomponents of capabilities and the two subcomponents of attitudes, the average valuation on knowledge *(K)*, capability *(C)* and attitude *(A)*, as well as traditional grade *(E)*.

ID	VKL	VKQ	VKS	VKC	VKU	VCT	VCA	VCC	VCM	VCP	VCR	VCS	VCU	VAS	VAE	K	C	A	EC	E
1	8,53	10,0	10,0	5,45	10,0	8,71	8,30	8,91	9,11	9,43	8,21	8,44	8,96	9,34	9,22	8,80	8,76	9,28	8,95	8,75
2	7,06	3,75	2,05	3,18	3,75	5,69	5,45	4,57	3,75	4,32	3,75	5,31	4,27	4,08	6,09	3,96	4,64	5,09	4,56	5,00
3	6,67	4,44	4,00	1,11	,00	4,40	4,44	3,81	3,60	3,50	3,64	3,64	4,44	2,22	7,50	3,24	3,93	4,86	4,01	4,00
4	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,00	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0
5	3,83	6,39	,00	4,44	5,00	3,30	4,44	4,64	3,80	5,00	4,43	5,00	6,94	3,89	5,21	3,93	4,70	4,55	4,39	4,50
6	4,17	3,75	3,75	3,06	3,75	5,00	5,83	5,24	4,50	5,00	3,75	4,89	3,75	5,14	4,79	3,69	4,74	4,97	4,47	5,00

Table 3: Some records in the database of results

Table 3 illustrates the possibilities of the evaluation model. Instead of a single data about each student, the standard grade *E*, we have a complete information on each competence level. For example, in the student with ID #5 has a grade of 4.50, typically assessed as insufficient. However, we can see that has an acceptable development of knowledge *Quantity* (VKQ=6.39) and *Uncertainty* (VKU=5.00). Similarly, he/she has a satisfactory level on *Problem posing and solving* (VCP = 5.00) and *Use of aids and tools* (VCU=6.94). On the other hand, the student with ID #6 has a classical grade satisfactory (E=5.00); however we see a lack in many components of the mathematical competence, since most of her scores are less than five. In both cases, we may recommend conducting some activities which had the objective of strengthening the shortfalls.

Conclusions

The assessment model we present has several advantages. We can highlight that it can be easily applied in educational contexts where it is difficult to make a more personalized assessment of the student, as may occur when the group is too big, or in a distance learning system. It is also possible to obtain not only a summative assessment, but also a formative evaluation designed to assess the individual student's level in each of the competences, setting minimum standards in each one of them to meet the objectives of the curriculum, identify gaps, to be able to recommend learning activities needed to achieve the desired level in each component of competence: knowledge, ability or others that we may consider.

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