Conceptualization – a necessity for effective learning of mathematics at school

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Abstract

This paper examined reasons why effective learning does not always materialize in mathematics and more specific in algebra at school level. In an attempt to identify possible reasons why effective learning evades learners a qualitative investigation was performed on students enrolled for mathematics education courses as well as on teachers furthering their studies in mathematics education. The outcomes were compared to possible reasons as portrayed in literature. In this paper the responses of the participants are discussed by analysing their responses to some of the questions posed to them.

Introduction

Teaching mathematics for effective learning was and still is a big challenge to mathematics teachers. Various reasons contribute to this phenomenon. It can be the way in which teachers execute their roles; it can be the confidence systems of learners based on their perspectives of mathematics; it can be the teaching methods used in the mathematics classroom; it can be misplaced outcomes; it can be the text books (incorrect content; way of unpacking the content; etc.) used to teach mathematics.

At first, effective learning and how it fits into the paradigm of social constructivism will be discussed. An investigation into the dichotomy between algorithms and heuristics; procedural knowledge and conceptual knowledge; inductive and deductive reasoning and concept definition and concept image will be address.

There is a saying that states that teachers must research their teaching and then teach what they have researched. This contribution can serve as an example of this saying.

Theoretical background

- **Effective learning**

De Corte and Weinert (1996) identified a series of characteristics of effective and meaningful learning processes which emerged from research that constitute building blocks that can serve as an educational learning theory. Those characteristics about which there is a rather broad consensus in the literature can be summarized in the following definition of learning:

> Learning is a constructive, cumulative, self-regulated, goal-directed, situated, collaborative, and individually different process of meaning construction and knowledge building (De Corte & Weinert 1996: 35-37).

The characteristics of this definition relates to the principles of constructivist teaching as discussed by Muijs and Reynolds (2005). It thus fits perfectly into the framework of reference of social constructivism.

Various authors (Cobb 1988; Hiebert & Wearne 1988; Nieuwoudt 1989; Schoenfeld 1988) stated that research has shown that teachers can formulate good goals, but despite of that there were still core problems that existed in the teaching of Mathematics at school. Learners are not seen by educators as constructors of their own knowledge; Learners cannot relate procedures of manipulating symbols with reality; Learners accept methods taught by educators without any criticism and apply it just like that; The over emphasizing of the answer; The teaching suppresses divergent thinking activities and creativity and problem-solving strategies are not established; are some of the core problems highlighted in the literature.

If this is true, then no effective learning took place if measured against the definition of effective learning by de Corte and Weinert. The dominant role of the teacher, the perspective of learners, misplaced objectives and the teaching methodology used to teach mathematics were identified by these authors as possible reasons that contributed to the existence of the mentioned core problems.

- **Algorithms and heuristics**

Researchers (Suydam 1980) distinguish between two methods of problem solving, namely the algorithmic and heuristic methods. An algorithm is defined as "...a recursive specification of a procedure by which a given type of problem can be solved in a finite number of mechanical steps" (Borowski & Borwein 1989:13). The aim of heuristic is to study the methods and rules of discovery and invention (Polya 1985:112-113). It is evident that self-discovery plays an important role in this method (Schultze 1982:44-45).

Heuristic methods are not rigid frameworks of fixed procedures which provide a guarantee for the obtaining of a solution. The purpose of value thereof lies mainly therein that you search purposefully and systematically for a solution (De Villiers 1986).

These two methods of problem solving clearly differ from one another. For instance, an algorithm ensures success if it is used correctly and also if the correct algorithm is selected and used. Algorithms are problem-specific, while the heuristic method is not problem-specific, because it is normally a combination of strategies. This leads to the fact that a heuristic method is applicable to all types of problems. A heuristic method provides the "road map", a blue print, which leads a person to the solution of a certain problem situation. In contrast to algorithms the heuristic method does not necessarily lead to immediate success (Krulik & Rudnick 1984).

It is important to note that, although the heuristic method could serve as guideline in the solution of relatively unknown problems, it cannot replace knowledge of subject content. Quite often the successful implementation of a heuristic strategy is based on the fixed foundations of subject-specific knowledge (Schoenfeld 1985).

The heuristic way of doing problem solving should play an ever-increasingly important role in the teaching learning situation where problem solving is the focus of teaching. Algorithms, on the other hand, form part of the subject content and are therefore also important. What is of cardinal importance, however, is that an algorithm should be part of the package of knowledge only after it was constructed in a heuristic manner.

Students will empower themselves if they are capable to apply a range of problem solving strategies when confronted with a problem (Schoenfeld 1988). The heuristic method should not be viewed as a goal in itself but must rather be seen as a way in which a certain goal is achieved. Drawing diagrams, for example, should not be taught as a unit in the mathematics classroom, but must rather be used to solve problems where applicable.

Groves and Stacey (1988) consider the strategies as important, especially at the beginning when actual problems are tackled. These strategies give the pupils a degree of control in the process of problem solving and it is important that they should be able to apply it spontaneously without being dependent on the teacher's support (see also Roux 2009).

- **Procedural and conceptual knowledge**

Students and even some teachers have a limited conceptual knowledge span of algebra and it was further found that there conceptual knowledge does not correlate with their procedural knowledge (O'Callaghan 1998; Hollar & Norwood 1999; Roux 2009). Procedural knowledge focuses on the development of skills and can it therefore be deduced that it relates more to the use and application of algorithms (O'Callaghan 1998). Conceptual knowledge on the other hand is characterised by knowledge that is rich in relationships between variables and also including the ability to convert between various forms of presenting functions, i.e. in table format or in graph format, etc. (Hiebert & Lefevre 1986). Conceptual knowledge lends it more to self discovery which relates more to the use of heuristics and inductive and deductive strategies.

Developmental, reinforcement, drill and practice as well as problem solving activities are generally used in the teaching and learning of mathematics (Troutman and Lichternberg 1995). Developmental and problem solving activities lends it more towards the development of conceptual knowledge whereas reinforcement and drill and practice activities lend it more towards the development of procedural knowledge. The advantage to first expose learners to developmental and problem solving activities is that they are challenged to develop conceptual knowledge before being exposed to procedural knowledge (Davis 2005).

RESEARCH QUESTIONS

The following research questions were investigated:

- Is the procedural knowledge which teachers use the outcome of conceptual knowledge?
- Is the procedural knowledge which student teachers use the outcome of conceptual knowledge?

RESEARCH METHODOLOGY

Qualitative research methods were used to find answers to the mentioned research questions. At first mathematics school textbooks and examination papers were analysed to determine to what extend the focus was placed on procedural and/or conceptual knowledge. A questionnaire consisting of mainly algebraic statements was designed based on these findings. These questionnaires were administered by the researcher. The target population consist of different groups of fourth year mathematics education students over the period 2005-2009 as well as practicing teachers who have enrolled for an advanced certificate in mathematics education and/or who have participated in mathematics education workshops (2005-2009). This was thus not a longitudinal study because each year different groups of students were involved in the study. The questionnaire consisted of ten questions. The respondents completed the questionnaire in class and it took them more or less 15 minutes to do so. The responses to each of the questions were either true or false. These responses were noted and the questionnaire was thereafter discussed and debated which contributed towards the reliability and validity of the questions posed in the questionnaire. During these discussions the researcher continually posed questions, obtained answers, and critiques the answers, to obtain a deeper understanding of the thought processes of the respondents.

RESULTS AND DISCUSSION

The outcome of two of the questions will briefly be discussed in the following paragraphs.

- Question 1: If $x^2 = 4$, then $x = 2$

Most of the respondents over the years indicated that $x = \pm 2$ should actually be the answer. The answer $x = 2$ was also indicated as the correct answer by quite a few participants. They arrived at this answer by substituting $x = 2$ into the equation and then found 4 as answer. It was also clear from the discussions that the procedure 'taking roots on both sides' was applied by most of the respondents in solving the equation. This procedure is also advocated by some text books. The participants did not realise that this procedure are actually lowering the grade of the equation from a quadratic equation to a linear equation and by doing that there can only be one answer, namely $x = 2$. The concept of solving $x^2 = 4$ was visualized by representing $y = x^2$(using excel) as a graph. The two values $x = \pm 2$ were identified as the solutions to the equation. The algebraic solution of the quadratic equation $x^2 = 4$ was discussed by solving the equation by means of factorization.

- Question 2: If $x^{\frac{2}{3}} = 4$, then $x = \pm 8$

Most of the respondents guessed the answer, but a few substituted $x = \pm 8$ into

$x^{\frac{2}{3}} = 4$ and concluded that the statement is true. In solving the equation algebraically – as is done in some text books - the statement was found to be true, but when represented as a graph, it was clear that $x = 8$ was the only solution. This once more was an indication that learners are in a framework of mind to follow procedures rather than conceptualise the problem. In question 1 they saw that the solving of the equation provide the solution and therefore argued that it must work in this instance as well. The solving of the equation does provide the solution if the restriction $x > 0$ is applied.

Over the mentioned period (2005-2009) only one student (mathematics on third year level) demonstrated a clear correlation between his procedural and conceptual knowledge. He applied algorithms where applicable but work heuristically when confronted with a situation that seemed unfamiliar to him. The majority of the students applied rules mechanically without reflecting on their answers. Rare evidence of conceptual knowledge was noted. It can thus be concluded that the students focused mainly on concept definitions and did not demonstrated concept images. Conceptualised knowledge was thus not the outcome of the application of procedural knowledge. The demonstration of subject content knowledge was not on a desired level and the same applied to their professional pedagogical knowledge.

The situation was even worse in the case of the teachers. Each group clearly demonstrated that they apply rules mechanically without applying problem solving strategies at all. They in other words did not work heuristically or inductively. A possible reason for this phenomenon could be that these teachers did not receive any training in mathematics at post grade 12 level. Their training in mathematical content was restricted to grade 12 level because they were all trained at Colleges of Education. Their mathematical factual knowledge was at a substandard. It is evident that they will not be able to apply their professional pedagogical knowledge in full when teaching mathematics due to the lack of mathematical content knowledge. It can thus also be concluded that conceptualised knowledge was not the outcome of the application of procedural knowledge for these groups of teachers.

CONCLUSION

Teachers and mathematics education students who participated in this research apply mainly algorithms when solving problems involving algebra. They work deductively and demonstrate procedural knowledge. It can be concluded that the same core problems discussed previously still exist in the teaching of mathematics and more particularly in the teaching of algebra.

It was evident that students and teachers who have participated in this endeavour have a limited conceptual knowledge span of algebra and it was further found that there conceptual knowledge is not in line with their procedural knowledge.

The questionnaire used in this investigation was not discussed in full in this written report because it is the intention to actively involve those who will attend this presentation by exposing them to the questionnaire. In the discussion of each of the questions the aspects as discussed above will be unpacked and debated.

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